Corporate Secular Stagnation: Bayesian Evidence on the Advanced Economy Investment Slowdown

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Abstract

Using a Bayesian mixed effects model on a large firm-level panel we show that the investment slowdown is a long-standing feature across 21 advanced economies since 1999. We find a flattening in the investment-q relationship over time, as firms become less responsive to investment opportunities. Weaker responsiveness to investment opportunities is closely linked to firms with more market power. A shortage of investment opportunities – as per the original secular stagnation thesis – predicts 40% of the variation in firms' estimated baseline investment rates.

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1 Introduction

Are advanced economies stuck in 'secular stagnation'? Perhaps no macroeconomic proposition has become more widely debated by economists today as they try to reverse slowing economic growth (Summers 2014, 2015). According to the 'secular stagnation' hypothesis, low investment spending has weakened aggregate demand, depressing growth and inflation, and with it the natural rate of interest (Eggertsson, Mehrotra, et al. 2019) and productivity growth (Ollivaud et al. 2018).

Testing this hypothesis requires analyzing firms' investment rates across advanced economies.[1] Yet there is remarkably little existing evidence on this. We know that U.S. firms' investment rates decline from around 2000 (Gutiérrez et al. 2017b; Fernald et al. 2017), but the slowdown is often believed to be more recent in Europe (Döttling et al. 2017).

As to the causes of the investment slowdown, the determinants are framed as being U.S. specific (Alexander et al. 2018; Philippon 2019). Chief among these is declining competition and a rise in monopoly power (McAdam et al. 2019; Eggertsson, Robbins, et al. 2021). Intangible assets are also advanced as a driver of the investment slowdown through it raising market power, creating measurement error, increasing external financing constraints, and reducing capital intensity (Farhi et al. 2018; Crouzet and J. Eberly 2019). Few firm-level studies on European investment rates exist, with no consensus as to the nature of their slowdown or the role of declining competition (Lewis et al. 2014; Döttling et al. 2017).

This paper asks whether secular stagnation in investment rates really is cross-country in nature. And if so what are its common defining features. We use a Bayesian 'mixed effects' (or 'hierarchical') model to help isolate the features of the slowdown which are shared across countries and years, and those which are not (Gelman, Carlin, et al. 2013; Meager 2019; Strauss et al. 2021).[2] All variables of interest are allowed to vary by country c (of which there are 21) and year t (of which there are 27). This helps deal with some endogeneity concerns (Roberts et al. 2013) and results in 189 parameters in the baseline model. The model uses a partial-pooling 'shrinkage' prior to ensure the number of parameters can scale with c and t without overfitting the data. We run the model on a Compustat dataset containing non-financial publicly listed firms incorporated in the U.S. and 20 other advanced economies between 1994-2020. Our unbalanced panel has 289,0964 observations on 30,439 unique firms.

[1] We define the investment rate as the ratio of capital expenditure to gross capital stock as the sum of gross property, plant, and equipment, intangible assets, and inventories.

[2] See Hsiao and Tahmiscioglu (1997) for an early application of mixed effects modelling to investment demand.

We use a standard cash flow-q investment model to estimate firms' investment demand (Tobin 1969; Hayashi 1982). Investment is a function of the investment opportunities facing the firm (represented by marginal q) and which we approximate using the average book-to-market value of the firm's assets. To this is added a cash flow rate variable to proxy for the potential external finance constraints facing the firm (Fazzari et al. 1988).[3] The third parameter of interest is the intercept, reflecting estimated baseline (mean) investment rates, since our predictors are mean centered. Our mixed effects empirical approach means that we do not have to assume that our three parameters of interest (q, cash flow, and the intercept) are constant over time – or by country (Erickson and Whited 2000). The time-varying intercept highlights exogenous shifts in the marginal product of capital and is often used to identify 'underinvestment' (Gutiérrez et al. 2017b).

Growing market power can be incorporated into a cash flow-q investment model through assuming decreasing returns to scale in the profit function (Cooper et al. 2001; Gomes 2001; Andrei et al. 2019). This represents a persistent (and potentially growing) source of measurement error (Erickson and Whited 2000) – driving a (further) wedge between unobservable marginal q and observed average q over time.[4] If firms' have growing market power this can, therefore, result in their investment demand becoming less responsive to observed average q over time (Andrei et al. 2019), captured in a flattening time-varying investment-q slope. A flattening investment-q slope over time is exactly what we find and provides an important proviso to the existing literature which frequently assumes that parameters of interest are constant over time and so cross-sections can be pooled (Erickson and Whited 2000; IMF 2015; Furceri et al. 2021).[5] Our study has six key findings:

i. Pooled raw investment rates decline secularly for advanced economy firms beginning in 1999. After 2001 median investment rates fall below 5% and stay there, declining further. This has been accompanied by stable and high profitability and declining investment opportunities (q values).

ii. Results from our 'cash flow-q' investment demand model show a much starker drop in firms' estimated baseline (mean) investment rates, beginning in 1999 and falling precipitously till the present, with a mild recovery between 2004-2008. Falling 'baseline' investment rates is one of two key features of 'underinvestment' by firms.

iii. Firms have become much less responsive to investment opportunities (flatter investment-q slope),

[3]For critique of this interpretation with respect to dynamic models see: Strebulaev et al. (2012).
[4]See Crouzet and J. C. Eberly (2021) for a recent model.
[5]Even though tests of constant slopes over time have been rejected in previous studies too: Erickson, Jiang, et al. 2014, p.219.

roughly halving in value between 1994 and 2020. This is robust to measurement error (Appendix B). This is the second major feature of 'underinvestment' and is correlated with the estimated decline in estimated baseline investment rates.

iv. We show empirically that declining investment responsiveness to q reflects growing firm-level market power. This finding is confirmed by several separate regressions which show a flatter q slope for firms with higher profitability, global sales market share, or rates of R&D expenditure.

v. Advanced economy firms are, overall, not financially constrained in our investment demand model, and global financing constraints have been declining since a 2010 peak 'cash-crunch'.

vi. Our extended regression model shows that, following the original secular stagnation thesis (Hansen 1939; Johnson 2022), firms are constrained by a relative shortage of investment opportunities. 40% of the variation over time in firms' estimated mean baseline investment rates is related to the non-financial corporate sector's net releasing of funds externally to shareholders, creditors, and bondholders, which increases considerably since 2000. Firms investing in net financial assets ('financialization' – Davis 2017), does not track the estimated investment decline. While retention of funds internally is only very weakly associated with the estimated investment decline, indicating that repairing over-leveraged balance sheets (Myers 1977; Koo 2011), or overcoming intangible asset induced financing constraints (Faulkender et al. 2019), is unlikely to have been a dominant factor behind the investment decline.

To our knowledge this paper is the first to show that the time-varying q coefficient is declining. This means that "underinvestment relative to q" (Gutiérrez et al. 2017b) really consists of two related but distinct phenomenon: declining estimated mean baseline investment rates *and* declining responsiveness to q. This finding is robust to using different priors, likelihoods, and estimation using non-Bayesian OLS.

We are also the first to provide preliminary cross-country evidence which connects declining q coefficients to firms' market power, after recent theoretical attention on this relationship (Andrei et al. 2019; Corhay et al. 2020; Crouzet and J. C. Eberly 2021). Such evidence supports the prevailing contention that falling investment rates reflect a lessening of market competition (Farhi et al. 2019; Crouzet and J. Eberly 2019), but indirectly through depressing firms' responsiveness to investment opportunities. Other interpretations given to a changing q slope relate to market distortions more generally (McLean et al. 2012; R. Chen et al. 2017), and the degree of private information contained in a stock (Q. Chen et al. 2007). Our time-varying q result shows that it is a mistake to view the depressing effects of market power on investment rates as just a U.S. phenomenon, since our time-varying q coefficients are estimated while accounting for country-specific effects in all parameters.[6]

Our finding that a dearth of investment opportunities constrains firms estimated baseline investment rates draws on related studies on *gross* corporate pay-outs (Lewellen et al. 2016; K. Kahle et al. 2021), though our measure is *net* payouts given that all public firms access external finance today (Farre-Mensa et al. 2016; Denis et al. 2018; Lian et al. 2019). Higher net releasing of funds is not a firm life cycle issue (Farre-Mensa et al. 2016), since firms of all sizes are increasing their net releasing of funds. This also highlights the absence of a finance constraint facing most firms. Gutiérrez et al. (2017b), using a similar net external financing demand measure to ours, show that firms with higher rated bonds are engaged in more net releasing of funds. Our specific finding on the absence of firm-level finance constraints supports Gutiérrez et al. (ibid.). Our hierarchical model uniquely shows that time-varying finance constraints have fallen since 2010 and so are unlikely to drive the continued decline in investment spending.

The structure of our paper is as follows: Section 2 details our cashflow-q investment model, dataset, and investment rate trends. Section 3 describes our Bayesian econometric model. Section 4 reports the model's findings. Section 5 extends the model to explain declining estimated baseline investment rates and discusses robustness and endogeneity. Section 6 concludes. The Appendix contains the econometric specification (Appendix A), measurement error model (Appendix B), and descriptive statistics of key variables over time (Appendix C). An online Appendix describes the dataset and variables, group predictors, and contains additional descriptive statistics.

2 Investment Model and Data

2.1 Cash Flow-q Investment Model

Below we sketch an intuitive cash flow-q investment model with market power. For a more detailed micro-founded model see Crouzet and J. C. Eberly (2021). Following the formulation in Lewellen et al. (2016), the value of the firm V_t is maximized with respect to the control variable investment I_t,

[6] It is also robust to inclusion of country-time interaction varying effects for all coefficients.

given the capital stock K_t in period t and subject to the net present value of its profits $\Pi(K_t, s_t)$, less adjustment costs related to investment $C(I_t, K_t, \lambda_t)$, and less investment expenditure I_t. Profits are a function of a state variable s_t, reflecting past investment decisions and the firm's capital stock K_t. Quadratic investment adjustment costs are related to an exogenous stochastic parameter λ_t. The recursive Hamilton-Jacobi-Bellman equation is:

$$V_t = \Pi(K_t, s_t) - I_t - C(I_t, K_t, \lambda_t) + \beta E_t[V_{t+1}]. \tag{1}$$

The first order condition (FOC) taken with respect to the control variable investment I_t in period t is (Romer 1996):

$$1 + C_I(I_t, K_t, \lambda_t) = \beta E_t[V_k(K_{t+1}, s_{t+1}, \lambda_{t+1})] \tag{2}$$

$$= q_t. \tag{3}$$

Equation 2 states that the firm invests until the purchase price of capital (fixed at 1), plus the marginal adjustment cost, equals the marginal value of capital. q_t is the present discounted value of future marginal revenue products of an additional unit of capital. This makes q the market value of an additional unit of capital. With a purchase price of capital fixed at 1, q is the ratio of the market value of an additional unit of capital to its replacement cost, if $\Pi(K_t, s_t)$ is homogeneous of degree 1 (Abel et al. 1994). We proxy q by the average book-to-market value of the firm.[7] Next, quadratic investment adjustment costs for $C(\cdot)$ are assumed. Substitution of this into the FOC leads to the following - with subscript I referring to the partial derivative with respect to investment:

$$\mathrm{C}_t = \frac{1}{2} a \left(\frac{I_t}{K_t} - \lambda_t \right)^2 K_t, \tag{4}$$

$$\mathrm{C}_I = a \left(\frac{I_t}{K_t} - \lambda_t \right), \tag{5}$$

$$\frac{I_t}{K_t} = -\frac{1}{a} + \frac{1}{a} q_t + \lambda_t, \tag{6}$$

where λ becomes the error term in the investment regression, a is a time-invariant adjustment cost parameter, and q_t is a sufficient statistic to explain the firm's investment rate. To get the firm's present cash flow into regression equation 6, assume that external finance is more costly than internal finance due to financial market imperfections, thereby creating a 'Pecking Order' of preferred sources of financing for the firm (Myers 1984; Myers and Majluf 1984). Assume that external financing demand

[7] We use market value of equity plus book value of debt for the numerator (market value) and total assets as the denominator (book value). This keeps the variable strictly positive, despite some loss of interpretation.

of the firm is roughly proportionate to $I_t/K_t > \Pi_t/K_t$, with quadratic external financing (EF) costs (Lewellen et al. 2016):[8]

$$\mathrm{EF}_t = \frac{1}{2} b \left(\frac{I_t}{K_t} - \frac{\Pi_t}{K_t} \right)^2 K_t, \tag{7}$$

$$\mathrm{EF}_I = b \left(\frac{I_t}{K_t} - \frac{\Pi_t}{K_t} \right). \tag{8}$$

The cost of external financing is assumed to be $b \geq 0$. Plugging the above into the Equation 1 leads to the following final regression specification which we estimate:

$$\frac{I_t}{K_t} = -\frac{1}{a+b} + \frac{1}{a+b} q_t + \frac{b}{a+b} \left(\frac{\Pi_t}{K_t} \right) + \frac{a}{a+b} \lambda_t. \tag{9}$$

Equation 9 estimates firms' investment demand schedule, with a slope of q in investment-q space. Cash flow Π_t/K_t enters directly into the regression equation; but we can see it will be of little significance if the cost of external finance $b \to 0$, or if the firm has no need to access external finance, such that $I_t/K_t < \Pi_t/K_t$. Lastly, to consider the impact of market power, assume the profit function is as follows:

$$\Pi(K_t, \theta_t) = \theta_t K_t^{\alpha} \tag{10}$$

If $\alpha < 1$, such that there is decreasing returns to scale in the profit function (Cooper et al. 2001; Gomes 2001; Andrei et al. 2019), then the Hayashi (1982) conditions are violated and marginal q diverges from average q, such that firms may no longer respond to average q in their investment spending decision.[9] As a result, a flatter estimated q regression slope we interpret as increasing market power, as in Andrei et al. (2019). [10]

2.2 Data

Below we describe the main features of our data. Further details on our sample and variable definitions are contained in the online Appendix.

Our sample covers non-financial publicly listed firms constructed through merging S&P's Compustat Global and Compustat North America databases. The data is consolidated at the firm-level. Our final sample consists of 289,0964 observations on 30,439 unique firms across 21 countries for the 27

[8] This is not fully equal to the amount of capital raised because it ignores adjustment costs.

[9] Though marginal q and average q can be proportional even in the presence of decreasing returns to scale (Abel et al. 1994; Andrei et al. 2019).

[10] This is consistent with the microfounded models in Andrei et al. (2019), whereby R&D spend by firms enhances their market power, both in the model with innovation jumps and in the model with learning.

years between 1994-2020. This includes all the major advanced economies plus the Cayman Islands and Bermuda, where an increasing number of advanced economy firms are legally incorporated. Country inclusion is first based on it having an average GDP per capita (nominal US$) of $20,000 or more between 1994-2020. The country then needs a minimum of 1,900 observations to be included to ensure a sufficient credible interval is found. U.S. incorporated firms comprise 36% of our sample and Japanese firms 19.6%. We use an unbalanced panel since a balanced design, with no gaps in observations for a firm between any two years, would exclude most of the largest firms in existence today and create considerable survivor bias.

Variable definitions differ somewhat by country based on differing accounting standards. The U.S. follows GAAP accounting standards, while the rest of the world largely follows IFRS.[11] Values are in nominal US$, converted into a common currency using the Compustat Global currency file. Our variables are reported gross, before amortization and depreciation, but after tax, unless stated otherwise.[12] Capital stock is our denominator for profitability and the investment rate. Capital stock is defined gross as the sum of gross property, plant and equipment (PPEGT in Compustat), intangible assets INTAN, and inventories INVT. Our findings are not dependant on the inclusion of intangibles (or inventory) in our capital stock measure.[13]

The investment rate is defined as the ratio of capital expenditure (CAPX in Compustat) to capital stock (PPEGT + INTAN + INVT. Profitability is defined as a firm's operating income before depreciation and amortization (OIBDP) less interest payments and tax costs. We use the firm's market-to-book ratio (MTB), calculated as the market value of the firm's *total assets* (equity plus debt) over the book value of these assets, as our proxy for Tobin's q. MTB likely captures average rather than margin q, with the two only equal under restrictive assumptions (Hayashi 1982). Use of MTB based on total assets, as opposed to MTB based only on the firm's *capital stock*, is motivated by the desire to ensure q remains strictly positive, since removing negative values would likely bias our dataset.[14] The

[11]Firms listed in Japan are not required to report using IFRS standards. GAAP and IFRS contain important differences in depreciation rules, implied by how development costs are capitalized, and also differences in how impairment losses and component depreciation are treated.

[12]Gross investment rates are recommended, rather than 'net', for cross-country comparisons (Lequiller et al. 2014).

[13]The BEA measure of capital stock now includes intangible assets (including software, R&D, and some intellectual property). Studies increasingly tend to include intangibles in their capital stock measure or at least adjust for it now (Fernald et al. 2017; Peters et al. 2017). See also: Haskel et al. (2018). Various methods to adjust intangible assets (which is measured net) to a gross measure can be undertaken but have not materially impacted other studies' results (Peters et al. 2017). Peters et al. (ibid.) notes the positive impact on q regression coefficient values from the inclusion of intangible assets in its calculation of capital stock.

[14]In theory, certain countries and types of firms are more likely to have negative MTB capital stock values (Damodaran 2013). In our particular sample, Japan contains a large portion of negative q values (as a book-to-market value of the firm's capital stock). A large portion of negative values also arise in 2008 with the financial crisis.

explanatory power of MTB is also roughly the same as other q measures (Erickson and Whited 2006, 2012). Lastly, we would expect q values to vary greatly depending on the accounting rules used by the firm regarding revaluation of the market value of PPEGT on the balance sheet,[15] as appears to be the case (online Appendix).

2.3 Initial Data Description

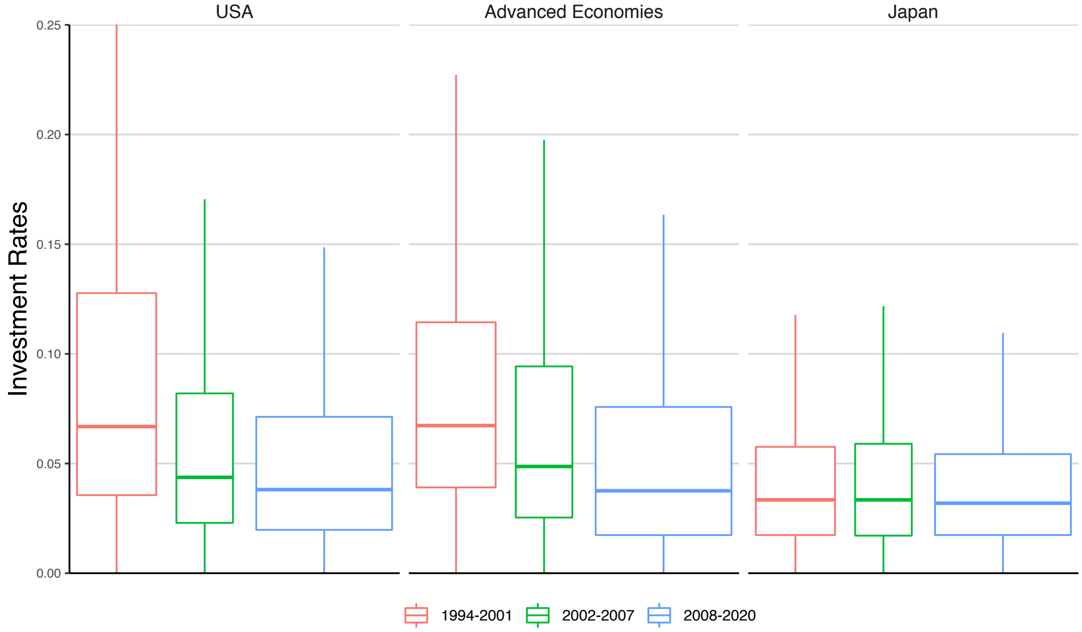
Below we plot firm-level investment rates over time by country grouping (Figure 1). Before describing this it is important to emphasize that our definition of investment (discussed above) differs from Gutiérrez et al. (2017b) for three major reasons, who adopt a non-traditional definition: (1) They include R&D with capital expenditure (CAPX); (2) they divide this by *total* assets; and (3) they use *net* capital expenditure (i.e. investment after depreciation). Our definition differs on all three accounts. For cross-country purposes in particular, *gross* investment is recommended rather than net (Lequiller et al. 2014). Use of gross assets as the denominator implies distortions if cash and short-term investments grow relative to total assets; and theoretically it does not capture the rate of growth of capital. Moreover, use of R&D plus CAPX to measure capital expenditure can involve double counting, especially for firms who use non-U.S. GAAP accounting which more easily permits capitalization of R&D expenditure.

Empirically, firm-level investment rates trends differ from those in national accounts, which show no secular decline in investment and instead a decline post-2008 (IMF 2015; European Investment Bank 2021). This reflects theoretical differences: National accounts now tend to include all business funded R&D expenditures as capital expenditure (Edworthy et al. 2007; Moylan et al. 2020), making it very different to how firm-level accounts treats R&D. R&D expenditures have increased in firm-level data too, but this still does not, on a quantitative basis, fully offset the decline in capital expenditure among U.S. firms, with their combination also peaking in 2000 (K. M. Kahle et al. 2017).[16]

[15]The ability to revalue assets to fair value under IFRS might create significant differences in the carrying value of assets as compared with U.S. GAAP (Gordon et al. 2008; PWC 2018). IFRS permits revaluation of these assets held on the book, while U.S. GAAP generally utilizes historical cost and prohibits revaluations of fixed capital. As a result a downward bias will be expected in book values (the denominator for our q proxy values) of U.S. GAAP firms. Compounding this is the fact that under U.S. GAAP, reversal of impairment is prohibited, while under IFRS it is permitted. We would expect then that q values would be much higher in the U.S. than in other advanced economies, only due to accounting reasons.

[16]R&D and capital expenditure are not interchangeable, especially from an accounting perspective. R&D has uncertain outcomes where capital assets have proven productive capacity beyond a year.

Figure 1. Firm-Level Investment Rates by Advanced Economy Group, 1994-2020



Note: *Showing box plots of firm-level investment rates. After 1998 and 2008 we see a structural drop. Year median is bold horizontal lines within each box. 'Advanced Economy' sample consists of firms incorporated in Australia, Bermuda, Canada, Switzerland, Cayman Islands, Germany, Denmark, Spain, Finland, France, Great Britain, Hong Kong, Israel, Italy, Netherlands, Norway, New Zealand, Singapore, and Sweden. 'Outliers' (observations outside of 1.5 IQR) not shown.*

Figure 1 shows the boxplot[17] of log investment rates of non-financial firms on our pooled sample between 1994-2020. The decline in investment rates occurs across advanced economies (Appendix Figure 8) and largely across firm sizes too (see online Appendix Figure 12). The timing of the shift in investment rates is in line with existing research on the U.S. (IMF 2015; Gutiérrez et al. 2017b; Alexander et al. 2018). Figure 1 helps clarify that Europe is undergoing a similar secular and long-running decline in investment rates (Caselli et al. 2010; Lewis et al. 2014; Döttling et al. 2017). [18] Japan's investment trend is unique showing a pre-existing continued stagnation during our sample time period, rather than a decline over time.

1999 and 2009 initiate large falls in investment rates following the Dot-com bubble and U.S. housing bubble bursting (Appendix, Figure 7). After 2001 pooled median investment rates among advanced economy firms fall below 5% and stay there. In 2020, during COVID-19, these median investment rates fall to a low of 2.8%. A very similar pattern of decline (with cycles) is evident for the upper hinge and whisker of the boxplot (which represent the 75th percentile of the sample and no further than 1.5

[17]For each box plot the lower and upper hinges correspond to the first and third quartiles (the 25th and 75th percentiles). The upper whisker extends from the hinge to the largest value no further than 1.5 x IQR from the hinge (where IQR is the inter-quartile range, or distance between the first and third quartiles). The lower whisker extends from the hinge to the smallest value at most 1.5 * IQR of the hinge. Data beyond the end of the whiskers are "outliers" and are plotted individually.

[18]Firms in Austria, Norway, Denmark, Spain, Switzerland, New Zealand, and the Cayman Islands stand out as having very high second investment peaks around 2006.

x the inter-quartile range from the hinge). This is also true for the bottom hinge (25th percentile of the sample). These generalised trends cast doubt on country-specific explanations for the investment slowdown.[19]

The general tendency of declining investment rates across advanced economies has been accompanied by very stable profitability along with a decline over time in pooled q values (investment opportunities).[20]

3 Econometric Model

In this section we detail the Bayesian hierarchical econometric model which we use to estimate our cash flow-q investment regressions. This is discussed further in the Appendix A.1.

A hierarchical model is a mixed effects model, combining fixed and random coefficients (Greene 2003; Hsiao 2014; Meager 2019; Strauss et al. 2021). This allows for the degree of heterogeneity to be estimated from the data rather than assumed *a priori* by the researcher. Fixed effects and pooling models are special cases of a hierarchical model (Gelman, Carlin, et al. 2013) that occur when the researcher sets the variance parameter for each country-varying or time-varying group to infinity (full pooling model, no differences), or to 0 (fixed effect dummy model, complete independence).[21] A hierarchical model instead estimates this variance parameter directly from the data. Country-varying and time-varying parameters are jointly estimated from a common distribution, each with their own variance parameter. This allows for a 'borrowing of strength' and information between country and year clusters (Tukey 1972), while still estimating the degree of heterogeneity between them (McElreath 2018).[22] This joint estimation approach produces a lower *total* mean squared error for the sum of the parameters within a group than a maximum likelihood estimator which estimates each parameter separately (W. James et al. 1961; Lehmann et al. 1998).[23]

[19]Such explanations include the outsourcing of labour-intensive production, low labour force participation rates, inflexible labour markets, and reduced government spending (Baldwin et al. 2014; Fernald et al. 2017; Alexander et al. 2018). While productivity growth has also slowed in Europe, labour force participation rates have increased across Europe, Canada, and Japan. Government spending in GDP shows uneven movements between 1995-2020 for the U.S., Japan, Korea, France, and the UK, and requires further investigation (OECD 2019a,b). See also: Ollivaud et al. (2018).

[20]By country grouping: over time q values have declined for pooled advanced economy firms (excluding U.S.) and become more compressed for U.S. firms (online Appendix C). This requires further exploration.

[21]For expositions of this model in commonly used panel econometric textbooks see, for example, Greene (2003) or Hsiao (2014).

[22]Put differently, the estimator regularizes estimates of the individual (random) effects towards the grand group mean, estimated from the data. There is more learned pooling – reflected by a small group-level standard deviation – when clusters within a group are similar to one another, or when a cluster has fewer observations. This helps ensure that countries or years with small sample sizes do not overfit their data (e.g. a fixed effect model), or that over-sampled countries or years do not dominate the inference (as in a pooled regression).

[23]A bias-variance trade off arises in this estimation as with most regularization estimators (G. James et al. 2013).

'Partial pooling' is particularly useful when using firm-level data. Firm nationality (however defined) is increasingly arbitrary and consolidated data cannot fully account for the fact that capital and profits are now also located in tax haven economies. In our sample in 2020, for example, Switzerland has the second highest median capital stock size among countries. While the Cayman Islands has more total capital stock incorporated there than all the firms in Canada and Norway combined. OLS won't be 'robust' to such outliers, while fixed effects methods (as practised) usually just control for intercept differences between countries. Partial pooling means that the estimates of firms' incorporated in Italy or Germany can 'learn' or 'borrow' from the estimates of firms in Switzerland or the Cayman Islands.

3.1 Bayesian Hierarchical Investment Regressions

Following the investment function from eq. 9, the firm's investment rate is determined by q and *cash flow* rates ('cash flow' for short), with the latter reflecting possible finance constraints facing the firm when sizable (Fazzari et al. 1988). Our hierarchical regression model focuses on the intercept of the investment demand function α, the slope of q, and the slope of *cash flow* (CF), by allowing for these (firm-level) coefficients to vary in their impact by year and country, in addition to being 'fixed'. This makes them our 3 'random effects'. Denoting $y_{c,t[i]}$ as the investment rate of firm i in country c and time t, the baseline regression is:

$$y_{c,t[i]} = (\alpha + \alpha_{c,t}) + \left(\beta^{q} + \beta^{q}_{c,t}\right)\mathrm{Q}_{c,t[i]} + \left(\beta^{cf} + \beta^{cf}_{c,t}\right)\mathrm{CF}_{c,t[i]} + \text{Controls} + \epsilon_{c,t[i]}. \tag{11}$$

$\mathrm{Q}_{c,t[i]}$ and $\mathrm{CF}_{c,t[i]}$ are the q and *cash flow* variables for firm i in country c and time t. Using them we estimate the 'fixed effects' population coefficients α, β^{q}, and β^{cf}. These 'fixed' coefficients represent the global 'average' intercept coefficient and global slope coefficients for q and *cash flow* for our total pooled sample. The 'random' effect counterparts to these variables are the coefficients $\alpha_{c,t}$, $\beta^{q}_{c,t}$, and $\beta^{cf}_{c,t}$ and have subscripts to indicate that they vary by country and year. They represent the intercept coefficient, and the q and *cash flow* slope coefficients for each of the 21 countries c and 27 years t, resulting in 189 parameters in the baseline model.[24]

The random effects coefficients estimate how each variable's impact, for a country or year, deviates from the coefficient's population average, such that $\beta^{q}_{c,t}$ shows how the impact of q on firms' investment

[24] This excludes group predictors added later. 21×3 random country effects, 27×3 random year effects, 3×3 variance parameters per year group and country group, 3×3 correlation parameters per year group and country group, 2 t-distribution parameters, 24 population-level predictors (including 10 size dummies and 11 sector dummies), and 1 AR error process coefficient.

rates in country c, or year t, deviates from the average impact taken across all countries or years. The extent of deviation within each group is captured in the group-level variance parameter, estimated from the data directly (rather than decided in advance by the researcher, as in a fixed effects or pooling model). *Controls* consist of $\gamma'^k \text{K} + \gamma'^{gics}\text{GICS}$, where K is 10 firm size bins (based on sales) and GICS is an 11 category industry dummy based on the contemporary Global Industry Classification Standard (GICS) codes. ϵ is an error term and we include an AR(1) error process to account for the panel nature of our data: $\epsilon_{c,t[i]} = \rho\epsilon_{c,t-1[i]} + u_{c,t[i]}$ where $u_{c,t[i]}$ follows a student-t error.[25] Appendix A.1 contains a more technical specification of our model.

Our random effects already effectively explore differences in financing constraints across firms in different years and countries. As a result we do not divide firms *a priori* into further groups, such as firm size, based on the degree of external financing constraints they might possibly face. Instead, we use firm size and industry as fixed effects control variables (Whited 1992; Hsiao and Tahmiscioglu 1997; Kaplan et al. 1997). Moreover, we do not find meaningful patterns in coefficients when estimating our random effects by firm size or industry. Only for robustness do we add a $country - year$ group across which all parameters can also vary.

Given our large sample size, we estimate our Bayesian model using variational inference (VI) (Gelman, Carlin, et al. 2013), instead of gradient-based Markov chain Monte Carlo (MCMC) (See Appendix A.3 for further details). VI saves considerable estimation time. Our results are largely identical when estimated using MCMC.

3.2 Hypotheses

Applying our Bayesian hierarchical ('mixed effects') model to estimate the 'cash flow-q' equation 11, we test the following three hypotheses on the causes and nature of the investment slowdown among advanced economy firms:

1. **Secular stagnation in baseline investment rates** (secularly declining *intercept* coefficients – $\alpha_{c,t} \downarrow$): The intercept of the investment demand curve is decreasing over time, reflecting exogenous declines in the marginal product of capital, all else being held equal. Because our data (and predictors) are mean-centred, this can be interpreted as a decline in firms' underlying mean impetus to invest.

[25] An AR(2) process did not improve the model fit by a relevant amount.

2. **Declining responsiveness to investment opportunities, reflecting firm-level market power** (declining q coefficients over time – $\beta_t^q \downarrow$): Firms are becoming less responsive to investment opportunities over time due to increasing market power of firms (Gutiérrez et al. 2017a, 2018).[26]

3. **Absence of external financing constraints** (negligible *cash flow* coefficients – $\beta_c^{cf} \rightarrow 0$): Firms are not financially constrained, due either to external financing becoming less costly and/or demand for external financing declining relative to fixed capital spending (Gutiérrez et al. 2017b; Döttling et al. 2017). We expect external financing constraints to also have a cyclical pattern over time in line with the global liquidity cycle (Rey 2019).

The hypothesis of "under-investment relative to q" (Gutiérrez et al. 2017b) is usually identified as only hypothesis 1 above (as q coefficients are held fixed). In practise, we show that under-investment in fact consists of hypothesis 1 and 2.

4 Results from Hierarchical Estimation of Cash Flow-*q* Model

4.1 Findings

Table 1 below reports the summary output from our hierarchical regression model with mean-centred predictors, a logged dependant variable, and logged q variable. Core findings are robust to measurement error (Appendix B). Model fit and convergence statistics are all as expected.[27] The Bayesian R^2, which summarises the model 'fit', is moderate at $[0.24, 0.28]$ for the 95% credible interval (Gelman, Goodrich, et al. 2019).[28] Adding a $country - year$ interaction group level, across which coefficients can also very, increases the model fit but at the cost of potentially overfitting.

Table 1 reports the fixed effects coefficients,[29] the variation in the random effect coefficients within each group (year and country σ), and the estimated correlation between the within-group random effects (ρ). For example, the variation in the random coefficients within each group c, t is measured by the standard deviation (SD) of that group's random effect, such that σ_{α_t} shows the variation in the random effect intercept between years. While the correlation between the estimated cash flow and q

[26] We test the market power hypothesis separately.

[27] Pareto-K diagnostic values often used when evaluating estimation results from variational inference (Yao et al. 2018). When Pareto-K is smaller than 0.7, it means that the computed variational posterior approximates the true posterior well. In our model, all Pareto k estimates are below 0.7.

[28] The autoregressive term substantially improves the R^2.

[29] Which double in our model as the global population effects for each variable around which the random effects vary.

Table 1. Summary of Hierarchical Model Regression Results

	Variable	Estimate	Est.Error	l-95% CI	u-95% CI
Fixed Effects	α	-3.12	0.01	-3.13	-3.11
	β^{cf}	-0.02	0.00	-0.03	-0.02
	β^{q}	0.14	0.00	0.14	0.15
Country Random Effects	σ_{α_c}	0.19	0.00	0.18	0.20
	$\sigma_{\beta_c^{cf}}$	0.07	0.00	0.06	0.08
	$\sigma_{\beta_c^{q}}$	0.04	0.00	0.04	0.05
	$\rho_{\alpha_c,\beta_c^{cf}}$	-0.01	0.06	-0.11	0.09
	$\rho_{\alpha_c,\beta_c^{q}}$	-0.09	0.08	-0.22	0.04
	$\rho_{\beta_c^{cf},\beta_c^{q}}$	0.02	0.10	-0.15	0.18
Year Random Effects	σ_{α_t}	0.18	0.00	0.17	0.18
	$\sigma_{\beta_t^{cf}}$	0.06	0.00	0.05	0.06
	$\sigma_{\beta_t^{q}}$	0.02	0.00	0.02	0.02
	$\rho_{\alpha_t,\beta_t^{cf}}$	-0.53	0.04	-0.59	-0.45
	$\rho_{\alpha_t,\beta_t^{q}}$	0.71	0.05	0.62	0.78
	$\rho_{\beta_t^{cf},\beta_t^{q}}$	-0.47	0.06	-0.57	-0.37
AR(1) Parameter	φ	0.69	0.00	0.69	0.70
Student-t Parameters	σ	0.53	0.00	0.52	0.53
	ν	2.55	0.02	2.53	2.57

Note: *Results are for Regression Model, equation 11. For each coefficient, the mean (estimate), standard deviation (Est.Err), and the 95% credible/uncertainty interval (l-95% CI and u-95% CI) are shown. l-95% CI and u-95% CI are 2.5% and 97.5% percentiles of the posterior distribution.*

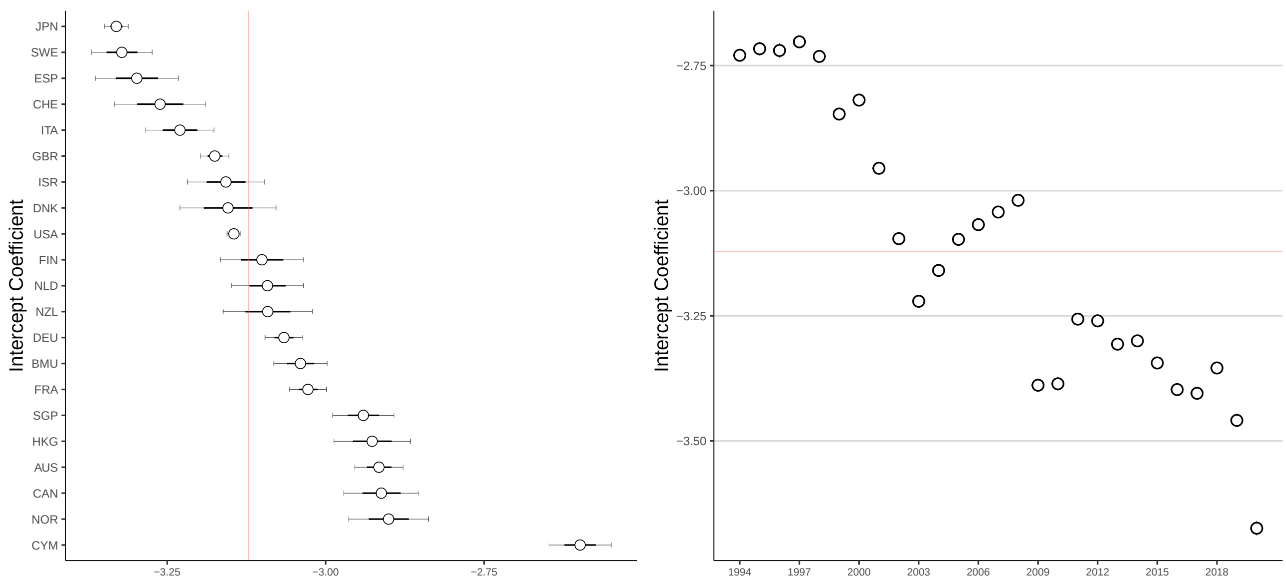
coefficients between countries is measured by $\rho_{\beta_c^{cf},\beta_c^{q}}$. Note that the estimated degree of the variation in our random effect coefficients (σ) within each group does not support using a pooled regression or a fixed effect estimator since variation in coefficients between years or countries exists but not to either extreme (of no differences or complete differences).[30] The model's findings are best explored graphically, as we do below.

Our findings confirm our three hypotheses. Firstly, Figure 2 shows a clear decline over time in baseline investment rates of firms, highlighting the secular nature of the investment slowdown. The

[30] A higher standard deviation coefficient, indicating larger estimated variation between countries (or years) in a coefficient's estimates, makes a pooled model, which estimates a single coefficient across countries or years, inappropriate. While as the standard deviation of the coefficient within each group increases, a no-pooling model (separate unrelated regressions) becomes more relevant, reflecting higher estimated variability in coefficients between countries and years. In general, our regression coefficients show greater variation between countries than between years, with higher estimated standard deviations for all country-group (random effect) coefficients, making a pooled model inappropriate for coefficients between countries in particular. Intercept coefficients show that largest degree of variation making them the best candidate for estimation using a no-pooling model (i.e. separate regressions for each country and year). However, the coefficient is still not large enough to justify this (Gelman and Hill 2006). Note also that the standard deviation for q by years $\sigma_{\beta_t^q}$ may seem the smallest at .02 but because q is in log units this reduces the size of the SD (Bland et al. 1996).

estimated random effect intercept coefficients by country and year is plotted, reflecting the estimated underlying mean impetus to invest, all else being held equal. We see a clear secular decline in the intercept of the investment demand curve over time beginning around 1998. The fixed effect (global) intercept is included in this figure (horizontal red line) but it's value is arbitrary, sensitive to changes in the dependent variables' units of measurement (as it is a log-level regression) (Wooldridge 2016, p. 37).

Figure 2. Intercept Coefficients by Country and Year, 1994-2020

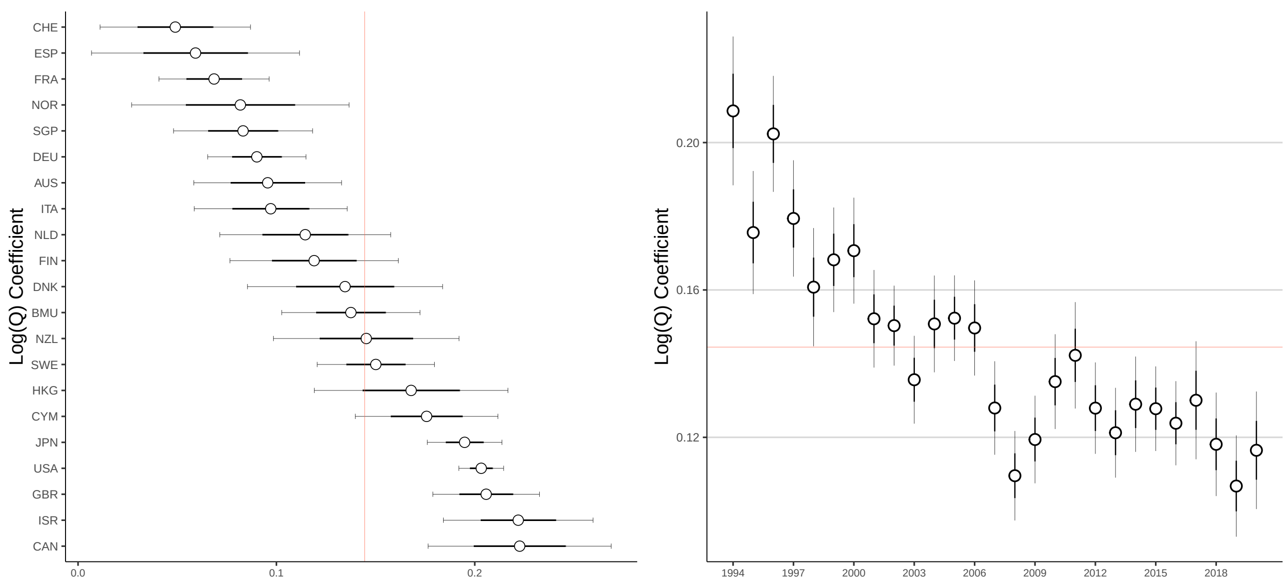


Note: *This shows the unexponentiated random intercept coefficient, i.e the predicted mean/median investment rate. The time trend of the intercept (right hand side graph) shows a clear secular downward trend marked by some cyclical fluctuations. The 68% random effect credible interval is shown in dark black, and the 95% random effects confidence interval in grey. Bayesian 95% random effect credible intervals display a high degree of certainty for clusters with larger sample sizes, this includes all years, 'USA' and 'GBR'.*

Unsurprisingly, despite partial pooling, the time-varying random effects intercept still follows fairly closely mean U.S. investment rates (K. M. Kahle et al. 2017), given its large contribution to our sample. Our time trend, however, bares no resemble to the (largely cyclical) dummy time effects estimated in Gutiérrez et al. (2017b) for U.S. firms (even after we add a measurement error correction to ours). This may be because of their radically different definition of investment, which is measured after depreciation and includes R&D spending over all assets. In Figure 2, firms from Australia ('AUS'), Canada ('CAN'), and Norway ('NOR') have the highest estimated baseline investment rates (intercept coefficients), in ascending order. Cayman Islands is a notable outlier (up) and highlights that country-level estimates might be pulled down if it was omitted. Japan ('JPN') has the lowest baseline investment rates, as might be expected given its persistently low investment rates in our sample. Compared to estimates from developing economy firms, these intercept coefficients tend to be lower (Strauss et al. 2021).

Secondly, Figure 3 (right hand side graph) shows a declining time trend for the estimated q coefficients, implying that firms are becoming less responsive to investment opportunities over time. We show later (Section 4.2) with additional regressions that this declining slope is probably due to growing firm-level market power (Gutiérrez et al. 2017a, 2018).[31] This reflects a growing wedge between average and marginal q over time (Eggertsson, Robbins, et al. 2018; Andrei et al. 2019).

Figure 3. Log q Coefficients by Country and Year, 1994-2020



Note: *Q coefficient shows a clear tendency to decline over time, with a strong cyclical component to it too. Log q coefficient is interpreted as an elasticity.*

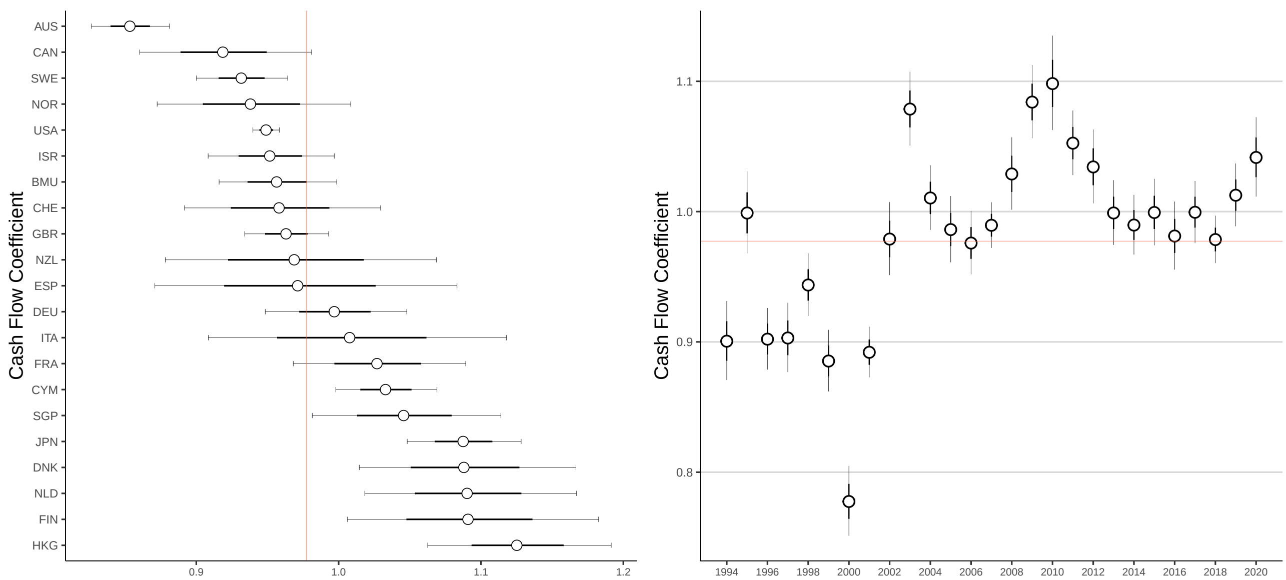
Also of interest is that q coefficients show much more variation by country than year (Table 1). Our q coefficient estimates are not directly comparable to previous studies since we use $\log(q)$ as our variable, but they are generally larger (Erickson and Whited 2000, 2012; Peters et al. 2017; Andrei et al. 2019). The 'fixed effect' value of $\log(q)$ is $= 0.14$ (Table 1), such that a 100% increase in q increases firms' investment rate by 14%, from an investment rate of say 5% to 5.7% (a 0.7 percentage point increase). These findings change somewhat when implementing measurement error correction in q (Appendix B). Our q coefficient estimate for U.S. firms, which previous studies focus on, at around 0.25, is in fact higher than for most other countries which we estimate (Figure 3, left hand side graph). While Canada, Israel, Great Britain, USA, and Japan appear to the most responsive to investment opportunities, with higher estimated q coefficients – though considerable uncertainty exists in the

[31]These interact measures of firm market power with q, all of which have a negative correlation with responsiveness to investment, as expected.

estimates for Israel and Canada.

Thirdly, advanced economy firms are not in general finance constrained (Figure 4). The right hand side graph of Figure 4 shows that finance constraints peak in (fiscal year) 2010, after which they decline, increasing during 2019 and 2020. Either way, external finance constraints are unlikely to contribute to the investment slowdown given that they are small in size – at least for publicly listed firms.[32] Posterior credible intervals are moderate for most countries and large for New Zealand (‘NZL’), Spain (‘ESP’), and Italy (‘ITA’), indicating a higher degree of uncertainty in the estimated impact of financing constraints for firms in these countries. Developing economy firms have somewhat higher cash flow coefficients (Strauss et al. 2021). Despite the high degree of uncertainty for country specific estimates (apart from the U.S.), cash flow coefficients are still negligible for advanced economy firms in most countries, especially after adjusting for measurement error in q (Appendix B).

Figure 4. Cash Flow Rate Coefficients by Country and Year, 1994-2020



Note: *Exponentiated fixed effect coefficients are the red lines at* 0.98 *(or -0.02 when not exponentiated). Total effect is shown here for country or year and is equal to the sum of fixed effect and random effect.*

Lastly, our model estimates from the data the correlation between variables within each group

[32] The exact size of the finance constraint can be calculated using the total cash flow effect for any particular country or year. This is equal to the sum of the cash flow fixed effect β^{cf}, and the relevant country or year random effect (β_c^{cf} or β_t^{cf}). This total effect, relative to the fixed effect (red line), is what is plotted in Figure 4. Since this regression relationship is log-level, we take the exponential of the cash flow coefficient to interpret it’s impact on investment rates. An exponentiated coefficient of above 1 implies a percentage increase in the geometric mean of y for a one unit (i.e. 100%) increase in cash flow rates; while a coefficient of below one implies a percentage decrease. U.S. firms have an economically insignificant total cash flow coefficient at less than 1 (left hand side graph of Figure 4), implying that, when cash flow rates are increased by 100%, the geometric mean of the investment rate, which is 5.2% in our sample, in fact decreases marginally. Sweden and Australia (and almost Great Britain) are the two other countries where the 95% posterior interval does not pass through 1. Other advanced economy firms, including tax haven firms, have small cash flow coefficients ranging from $1.0 - 1.2$, implying that when their cash flow rates increase by 100%, their investment rates increase from 5.2% to between 5.2% – 6.24% (or an increase of 1.04 percentage points at most).

using the LKJ prior (as reported in Table 1). Unsurprisingly, significant correlations exist at the time dimension. The strongest is between q and the intercept $\rho_{\alpha_t,\beta_t^q}$ at $[0.62, 0.78]$. This could mean many things, including a common exogenous factor driving both down, and requires further research.

The overall results on the time-varying pattern of the intercept, q and cashflow coefficients are farily robust to estimation using OLS with interaction effects,inclusion of a $country - year$ group by which coefficients vary, using alternative likelihood functions and different priors (including a much looser LKJ prior on our covariance structure), adding a measurement error model on top of our model, and estimation using MCMC instead of VI.

4.2 Q and Market Power

Below we support our interpretation of the declining q coefficient reflecting growing firm-level market power. Historically, the relationship between market power and q was assessed through investigating if raw q values were sustained above 1 (Lindenberg et al. 1981; Smirlock et al. 1984; Salinger 1984).[33] We instead focus on the slope coefficient of q (see Appendix B for model which accounts for measurement error). Our interpretation of the slope of the q coefficient is inspired by the micro-founded models in Andrei et al. (2019), whereby successful R&D yields market power. Andrei et al. (ibid.) finds that the slope of the investment-q regression decreases with market power, both in a model with innovation jumps and in the model with learning. We try to empirically ground the intuition developed by Andrei et al. (ibid.), that a firm's market power manifests in a flatter q slope using the regressions below.

We run three different specifications, each of which proxies for market power depressing the effect of q on investment. Following Q. Chen et al. (2007), we interact our market power proxy variables with q to see if our interpretation of q has an empirical basis. For our first proxy of a firm's market power we calculate their global sales share within a given GICS sector, after excluding all remaining real estate and financial firms. In keeping with our emphasis on global time-varying effects, we use the firm's global sales share instead of it's share within a given country (De Loecker et al. 2021). Thus, we assume that the relevant domain over which a firm competes and asserts market power is at the global product level. The global sales share is calculated as a firms global product's sales relative to total sales in a given sector and year, globally, and converted into a percentage. It is well known

[33] The idea being that, in theory, existing firms should expand and/or new firms should enter the market to take opportunity of such elevated market-to-book values. Sustained q values, therefore, implies a degree of barriers to entry and/or market power by incumbents to allow for this to persist. Under mismeasurement of q, q values > 1 became more likely, since book values are not measured properly if they contain large amounts of intangible capital, for example. Though further arguments are required to note that q might have a time-bias.

that industry concentration measures constructed with Compustat data have considerable limitations though (Ali et al. 2008) – and which we are unable to overcome within the confines of this paper.[34]

For our second proxy of market power, we interact the firm's profit rate with q. Our measure is similar to a Lerner Index except it is not a 'margin' since we divide the profit by capital instead of sales (though our results are invariant to doing so). Profit rate is a crude yet useful measure of a firm's market power.[35]

Lastly, in-line with the models of Andrei et al. (2019), R&D (XRD in Compustat) spend can provide a firm with durable market power. We interact R&D spend rate (R&D over capital stock) with q and also add a R&D standalone variable.Note that for R&D variable all NA values are replaced by zero. The results of these regressions are shown below in Table 2 below.

Table 2. Market Power and q Interaction Regression

Model	Interaction Coefficient	SD
Product Sales Share	- 0.065	0.008
Profitability	- 0.085	0.003
Rate of R&D	- 0.12	0.06

Note: *Results are for the same regression model as in equation 11, with log investment rate as the dependant variable, but with an interaction term with q added. Estimated using Bayesian variational inference. For R&D regression, we also include XRD/capital as a fixed effect, in addition to interacting it with q. Profit share regression filters out all firms with negative profits. Results are robust to estimation using OLS, which includes unrelated interaction effects for the random effects, instead of Bayesian random effects estimated from a joint distribution for all countries and another for all years.*

All regressions have have expected negative signs in their interaction effects with q and have reasonably sized posterior credible intervals. This indicates that, as a firm's market power increases, its tendency to respond to investment opportunities declines. The R&D regression has the largest coefficient effect size but also the largest credible interval, reflecting the fact that around 20% of the XRD observations are still very small in size. Results are fairly robust – though interaction effects are

[34]Industry concentration measures are not always adjusted for imports (Gutiérrez et al. 2017a) and common ownership (Schmalz 2018). Moreover, the relevant market at which firms compete is complex and impossible to define using highly aggregate sectoral data (Autor et al. 2020; Basu 2019). In addition, they are based on price-based measures of concentration (whereas major consumer products are increasingly provided for free, e.g. Google Maps); and cannot say anything directly about the degree of competition or market power under product differentiation (Capobianco 2018; De Loecker et al. 2021). Notes De Loecker et al. (2021):"As long as firms compete according to Cournot quantity competition, and produce identical products, the HHI is a direct indicator of market power. However, this holds only as long as products are perceived as identical to all consumers." When products are differentiated, there is no longer a relation between concentration and market power.

[35]We could focus on profitability within specific industries or markets. Different industries and product markets will yield different returns on capital (Bork et al. 2013). But as with attaining accurate sales concentration measures, a key challenge is that no clearly defined market can be undertaken using Compustat data alone within which a firm might have market power. For profit rate's connection to mark-ups see Basu (2019).

notoriously sensitive to estimate (Gelman 2019).[36]

5 Secular Stagnation and Uses of Funds

How to explain the persistent global decline in estimated baseline investment rates (time-varying intercept coefficients)? In this section we use these coefficients as additional data to be explained by group-predictors from the cash flow identity. We extend our baseline model to estimate if declining baseline investment rates is linked to non-capital uses of funds: pre-cautionary savings, non-capital investments & acquisitions, or a shortage of investment opportunities (net releasing of funds externally). If higher pre-cautionary savings or non-capital investments is associated with the decline in estimated baseline investment rates then this implies a crowding out of investment. While if instead the decline is linked to a net releasing of funds, then this shortage of investment opportunities implies deeper macroeconomics factors at work depressing investment behaviour, such as population ageing and slowing innovation (Hansen 1939; Johnson 2022).

5.1 Uses of Funds: If not CAPX then what?

Deeper explanations of the estimated decline in time-varying intercept coefficients (estimated baseline investment rates) are beyond the scope of this paper. We instead take firms' uses / sources of funds from the cash flow identity, and analyze which use of cash flow (after deducting for capital expenditures) is most closely correlated to the estimated variation in the time-varying intercept of the cashflow-q investment model. Crudely put, declining capital expenditure has seen an increase in funds available to the firm for other activities. This is because profitability has remained consistently high. The direction of causality, however, from use of funds to capital expenditure (CAPX), or vice versa, we cannot establish here. But we can establish which use of funds by the corporate sector as a whole co-moves with the estimated decline in intercept coefficients between years. Using the cash flow

[36] For profitability, similar results occur for using profit share within a given *gsector* (though not as strong). It is not robust to inclusion of firms with negative profit rates of which a large portion of our sample has, but econometrically this is unsurprisingly. For the sales share / concentration regression, if instead we use a firm's gsector's sale share within its country of incorporation, the coefficient size declines in size but is still the right sign. If instead we use the Herfindahl–Hirschman Index for calculations, coefficient is of correct sign but smaller in magnitude. Profit rate is robust to use of different denominators, including sales. (In this instance we also use profit over sales as the global fixed effect and the random effect which varies by country and time.) For our R&D rate regression, it is not robust to using different denominators and the coefficient changes when using the log of the variable. XRD over the sales variable (SALES in Compustat) shows a wide credible interval passing through zero $[-0.014, 0.027]$. Only when XRD over sales is logged does the coefficient's credible interval remain negative. When logging the XRD rate over the capital stock the variable's sign changes. This reflects the fact that the variable has a large concentration of small values with a long right tail. A log regression recognises this variation more keenly and so leads to less clear estimates.

identity from the firm, whereby Operating Cash Flow (OANCF) + Investing Cash Flow (IVNCF) + Financing Cash Flow (FINCF) = Cash and Cash Equivalents (CHECH), we have:[37]

$$\text{(OANCF - CAPX)} = \text{(CHECH) - (IVNCF + CAPX) - (FINCF)}.$$

If the variable takes on a negative value then this implies a use of funds by the firm, while a positive value is a source of funds. This identity highlights three channels through which operating cash flow (OANCF) can be used, after deducting for capital expenditure. It can be: (1) Left to accumulate within the firm as cash (CHECH); (2) Used for non-capital investments (IVNCF excluding CAPX),[38] which includes acquisitions; or (3) Released externally in net to creditors, debtors, and shareholders (FINCF). Based on this cash flow identity, we distinguish between three broad arguments about the relationship between the decline in CAPX and firm's uses of 'free' cash flow:

1. **Precautionary savings** (CHECH). Firms are building up cash piles, targeting a net change in the stock of cash and equivalents which they hold – and adjust all uses of cash flow, including CAPX, accordingly. Precautionary savings may be a response to the 2000 dot-com bubble or 2008 GFC as firms deleverage (Koo 2011); or reflect saving due to financial market imperfections arising from not easily collateralizable intangible assets (Caggese et al. 2017; Faulkender et al. 2019).

2. **Investing in the stock market and acquisitions – 'financialization'** (IVNCF removing CAPX). Instead of devoting funds to CAPX, firms are using their funds for non-fixed capital investments, which may have a higher return.[39] This includes short-term investments made by the firm as well as acquisitions. We call this the 'financialization' hypothesis (Davis 2017), though it is consistent with a simple portfolio allocation model based on different relative returns to fixed and financial assets (Foley et al. 1970).

3. **Shortage of investment opportunities** compared to robust cash flow (FINCF). We proxy this using the net external financing demand of the firm, which indicates if the firm is a net external releaser of funds (to shareholders, creditors, and bondholders) or a net external borrower of funds – after taking into account equity issuance and purchases, dividends, and all borrowing

[37] We exclude exchange rate effects (EXRE) which will mean the equation will not perfectly balance for most firms.

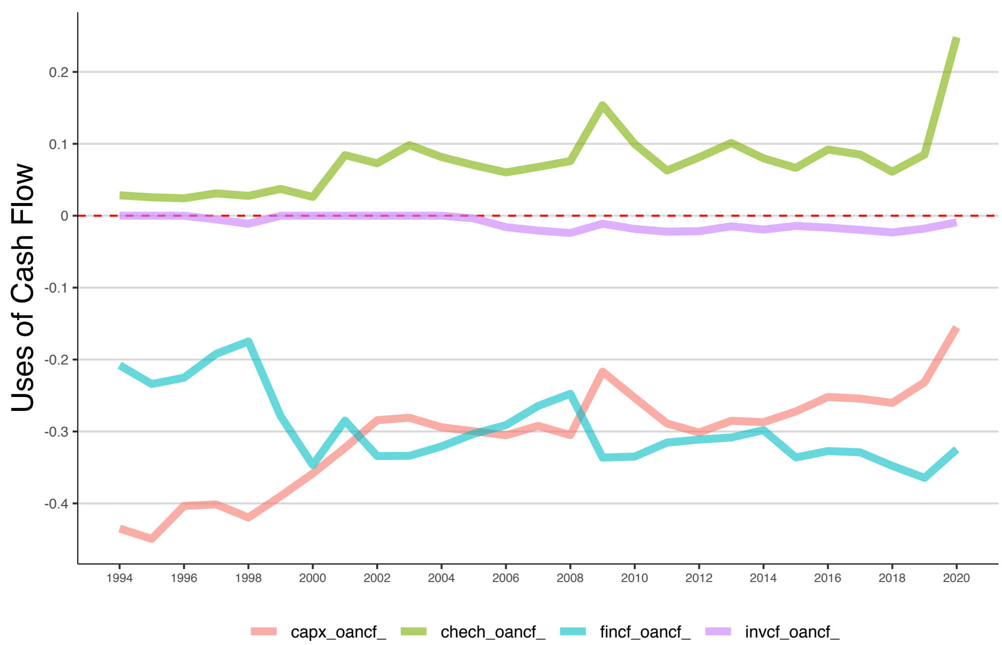
[38] This means adding back CAPX to IVNCF, which already includes, i.e. deducts, CAPX from it previously. As a result adding it now effectively removes CAPX from it.

[39] One cannot calculate this properly in Compustat due to data limitations.

and lending (see online Appendix C.3 for full definition).[40] The net releasing of funds likely reflects weak expectations for sales demand growth (Benigno et al. 2018; Klein et al. 2020) – see also Appendix C. The causes of this may be numerous including ageing and slowing innovation, as in the original secular stagnation hypothesis (Hansen 1939).

Figure 5 shows median cash flow uses and sources (relative to operating cash flow) for our pooled sample. A negative value implies a use of funds and a positive a source of funds. We see that median CAPX out of operating cash flow falls from -44% in 1994 to a low of -15% by 2020.

Figure 5. Uses and sources of cash flow relative to Operating Cash Flow, 1994-2020



Note: *Showing median values of uses of operating cash flow (Compustat OANCF), 1994-2020. Negative implies a use of funds and positive a source of funds. Declining CAPX out of operating cash flow, therefore, is reflected in this ratio becoming less negative.*

Declining (CAPX/OANCF) has been accompanied, first and foremost, by a greater portion of operating cash flow being released in net externally to shareholders, creditors, and debtors, from around -20% to a low of 36% in 2019 (FINCF/OANCF). We show in the online Appendix C.3 that growing net releasing of funds reflects the fact that firms increasingly have a surplus of available financing relative to declining or stagnant investment opportunities.[41] The other notable tendency has been for the firm

[40] It records almost all cash inflows and outflows between the firm and its external creditors, bondholders, and shareholders. It excludes dividend received and interest payments on debt, however. FINCF is similar to the net external financing measure used in Gutiérrez et al. (2017b), drawing on Frank et al. (2008), but includes short-term borrowing.

[41] There is a clear tendency, since 2000 especially, for firms and the corporate sector as a whole to increasingly become net external releasers of funds (online Appendix Figure 14). This implies a negative net external money demand, reflecting a shortage of investment opportunities relative to cash flow (Appendix C). These findings are not driven by a decline in new listings on the U.S. public exchanges (online Appendix C.3). Since, although older firms with fewer investment opportunities do release more financing externally (Brealey et al. 2011), the increased tendency to release occurs across firms of all sizes and industries (cf. with Jensen 1989).

to accumulate more cash out of operating cash flow, increasing from around 2% in 1994 to around 25% by 2020 (CHECH/OANCF), though its pre-COVID level hovered closer to 8% on average. Finally, operating cash flow devoted to non-capital investment activities has grown but remains low (IVNCF + CAPX/OANCF). These trends remain when U.S. or Japanese firms (subject to greater cash flow measurement error) are removed from our sample.

5.2 Extended Econometric Model with Group Predictors

Our hierarchical model is extended to also (concurrently) predict the mean of the intercept coefficient distribution M_β^α for each year group t (Gelman, Krantz, et al. 1999; Gelman and Hill 2006):[42] μ is estimated for the year group t only, using as data the number of estimated intercept coefficients within the year group, such that it runs from 1 to 27:

$$\beta_t \sim \mathrm{MVN}(M_\beta^\alpha, \Sigma_{\beta_t}) \tag{12}$$

$$M_\beta^\alpha \sim \mathrm{N}(\gamma_0 + \gamma_1 \mu, \sigma_\alpha). \tag{13}$$

Table 3 summarizes the regression output of the extended hierarchical model, each containing a different one of the three cash flow statement group-level predictors from Figure 5. This tests the hypotheses initially stated. Our focus is on whether the macroeconomic (group-level) predictors reduce the unexplained variation in the standard deviation of the time-varying intercept coefficients, SD(Intercept$_t$) $= \sigma_{\alpha_t}$. Results are not sensitive to changing the denominator of the predictors.

Table 3. Group Predictors Account for Variation in Time-Varying Intercepts

	Baseline Model		**GP 1: FINCF**		**GP 2: CHECH**		**GP 3: IVNCF**	
Variable	Est.	Est.Err	Est.	Est.Err	Est.	Est.Err	Est.	Est.Err
σ_{α_t}	0.139	(0.001)	0.084	(0.001)	0.124	(0.002)	0.143	(0.002)

Note: *Showing the estimated standard deviation of the time-varying intercept coefficients, from the baseline regression model and with group predictors added separately. All group predictors are normalized by OANCF after trimming top and bottom 0.1% of observations. IVNCF excludes CAPX (capital) expenditures. FINCF group predictors as GP1 are especially effective in reducing unexplained variation in the time-varying intercepts (σ_{α_t}), as the SD declines the most there. Regression run only on years 1994-2019 due to 2020 being an outlier. Coefficient values and signs of the group predictors are as expected (not shown).*

Table 3 shows that, at the macroeconomic level, the investment slowdown does not appear to be

[42] In a classical regression the group-level coefficients (as data) and the group-level predictors would be collinear, and instead must be run as two separate regressions, as in Hsiao and Tahmiscioglu (1997). This problem is avoided in a Bayesian model because of the partial pooling of the random group-level coefficients toward the group-level linear model. Adding predictors at the group level in a multilevel model corresponds to the classical method of contrasts in the analysis of variance (Gelman and Hill 2006, p. 497). Group-level predictors are often interpreted as 'contextual effect' in the social sciences.

driven by an increase in precautionary savings (CHECH). Out of the three group predictors, representing different uses of cash flow (based on the above cash flow identity), only a shortage of investment opportunities (FINCF) has good explanatory power in predicting the variation in our time-varying intercept.[43] CHECH and IVNCF have little to no predictive power, respectively, as can be seen by the "est." standard deviation coefficient declining by little or increasing.

The *median* corporate sector's external financing balance (net releasing) within a given year (FINCF), accounts for 40% of the variation in the time-varying intercept, reflected in the standard deviation of the coefficient declining from 0.14 to 0.084, while uncertainty in the estimates remains roughly the same (Table 3). Using the proportion of firms who become net external releasers of funds (in a given year) as a robustness group predictor accounts for a similar portion of the variation. Causality cannot be determined from this though since we are using accounting identities.[44]

5.3 Robustness and Measurement Error Model

We undertake a barrage of tests and alternative specifications to ensure that our model results are not sensitive to any single aspect of our model and that our model properly reflects the true variation in the data (Strauss et al. 2021). We re-estimate our model using OLS, using interaction effects (slope dummies), and find no major qualitative differences in time trends or results. Some differences in OLS exist as expected, largely due to the partial pooling process in our model which moderates more extreme estimates by year or country. MCMC estimation of the Bayesian model does not change the results as long as similarly loose priors are used. Our student-t likelihood performs somewhat better than a normal likelihood in predicting certain parts of out data. It is preferred though largely because it is robust to outliers due to its longer tail. It also effectively adjusts for a particular model of heteroskedastic normal errors in our context (Arnold 2019). Heteroskedasticity is modelled explicitly through our varying-coefficients model, with each group in our model having its own variance parameter, in addition to a common pooled (firm-level) error (Gelman and Hill 2006). Our log-log specification for the investment-q relation in our model further reduces heteroskedasticity dramatically.[45] Despite

[43] It is important to divide by OANCF instead of SALES which is not as well behaved at the median.

[44] Note that there can be no presumption that this net releasing of funds is a poor use of capital, given that we have shown that firms are not financially constrained in general and remain responsive to investment opportunities. Instead, if saddled with too much 'free cash flow', committing to its release can be optimal to avoid managers taking up poor investment opportunities (Jensen 1986; Tirole 2010). This is consistent with the capital structure literature's findings on the implications of Pecking Order and Agency Theories for firms' gross distributions (Fama et al. 2002).

[45] This can be seen by running simple quantile investment regressions of q on investment and plotting the fits across quantiles (Koenker et al. 2001).

a large dataset, different priors can have a considerable impact on our coefficient estimates, especially those which are too informative. This reflects the complex nature of our model and is made worse the more groups are added across which parameters can vary.

Our group-level structure by year and country is important to ensure variation in the data is not misassigned to a different 'level'. This occurs when one of these groups are omitted from the model, or if only a $country - year$ group is used (Schmidt-Catran et al. 2015). Using an 'industry' or 'firm size' group across which coefficients can vary produces results with no notable trends between firms of different sizes or industry – though some trends are evident in financing constraints by firm size, as is to be expected. A country:year group j, with 21 x 27 = 567 clusters, is added for robustness, produced by interacting the country and year groups together. This serves largely as a control group to ensure that country-specific time effects are not driving any of the results. Including this group does not change the core results but does make the model far more sensitive to prior specifications and is less robust to adding a measurement error model on top of it – probably due to the added computational complexity.

Most robustness measures for cash flow-q papers concern endogeneity.When data has a natural clustering, especially with a time component, modelling the clustering directly helps correct for the unobserved correlation which occurs from unobserved shocks correlated within a country, a year, or a specific year within a country (Wooldridge 2010, chapter. 20).In our case, our model has one error term common to all firms within each group of the model (plus the pooled regression level error), such that it assumes a degree of correlation across all firms within the same year; across all firms within the same country; and across all firms within the same year for a country.[46] Put differently, our model best deals with unobserved heterogeneity by directly modelling it through allowing slopes and intercepts to vary across country and year clusters. Ignoring parameter heterogeneities among cross-sectional or time-varying units, and instead estimating pooled average coefficients, can lead to non-sensical parameter estimates if the averages have no representativity across individual countries or years (Hsiao 2014).

Endogeneity is a particular concern in firm-level regression for several reasons – especially when

[46]This induces correlation in our $n \times n$ covariance matrix for the error terms (Gelman and Hill 2006). This is modelled explicitly by giving all firms within the same country, the same year, or same $country - year$ cluster a common error term.

the dependant variable is the investment rate. But dealing with it is not as simple as usually posed.[47] We include a lagged AR(1) error structure to deal with serial correlation and the dynamic structure of the model. This is measured very precisely in our model, with a 95% credible interval of $[0.69, 0.70]$. Adding a second autoregressive error does not improve model fit much, has a small coefficient, and adds significant computational time.

Endogeneity in cash flow-q regressions are endemic especially if q is mismeasured (Erickson and Whited 2012, 2006). Model estimation must take this into account (Whited 1992). In such cases q might not fully capture firms' future investment opportunities, due to the firm being young with limited information, or if q is measured with error due to average q diverging from marginal q (perhaps to the firm's market power). Measurement error in q not only biases downward q coefficients but complicates the interpretation of cash flow as measuring the cost of external financing. For if q is mismeasured, and if cash flow also reflects unobserved future investment prospects, then cash flow may impact investment (the dependant variable) only because, like with mismeasured q, it is correlated with the 'true' (perfectly measured) marginal q used by firms to make their investment decisions. Correcting for measurement error in q is, therefore, also vital in ensuring that cash flow can be measured and interpreted properly. Note that this impact persists even if the correlation is negative.

A Bayesian approach to measurement error is formulated by treating the true quantities being measured as missing data (Clayton 1992; Richardson et al. 1993; Gelman, Carlin, et al. 2013). The full model is detailed in Appendix B. This Bayesian approach to measurement error is computationally demanding but has several advantages over other traditional methods. Firstly, the Bayesian estimator provides a posterior distribution that takes into account uncertainty due to estimating other parameters; while the classical estimator corrected for attenuation bias would require bootstrapping – or some type of asymptotic approximation – to account for this uncertainty. Secondly, Bayesian inference averages over plausible values of mismeasured q in light of the data, rather than imputing a single best-guess and then moving forward with the estimation as if this guess is correct. Uncertainty in estimation of q is instead propagated forward in a Bayesian approach. Thirdly, we can integrate the measurement error model while keeping most of our model complexities, including the mixed ef-

[47] For one, right hand side variables in such regressions are invariably endogenous and an identification problem arises which can never be properly resolved due to the simultaneity present. Adding lagged dependant variables does nothing to deal with simultaneity bias, even though it is commonly believed to (Reed 2015). Adding a lagged dependant variable may control for other types of endogeneity and can help deal with serial correlation though (Hsiao and Tahmiscioglu 1997). Lagging the dependant variable, however, has significant data requirements and introduces other forms of survivor bias, excluding firms who do not conform to the balance panel (such as newer established firms).

fects structure, the autoregressive error structure, student-t likelihood, and other deviations from a simplistic panel regression model (Carroll et al. 2006).

We conduct a sensitivity analysis in Appendix B to explore at what levels of measurement error our model results might change. At low to medium levels of measurement error our cash flow coefficients declines even further, indicating that some of the cash flow coefficient is due to the correlation with q.[48] This indicates that financing constraints are likely even less significant than those detailed in the model results above. Even assuming very large levels of measurement error in our sensitivity analysis does not change the time-varying pattern of q change or our intercept coefficient. Credible intervals do increase notably though.

6 Conclusion and Discussion

Using a large dataset of Compustat firms we provide evidence across 21 advances economies that 'secular stagnation' in investment rates between 1994-2020 exists and is a cross-country phenomenon consisting of two primary components. The first is a strong and persistent decline in baseline (mean) investment rates among advanced economy firms since 1999 – approximated by the time-varying intercept of firms' investment demand. It falls even further during COVID-19 in 2020, highlighting the potential scarring which the pandemic may leave on the economy.

'Underinvestment' has a second major component to it, conspicuously absent from the existing investment literature (Gutiérrez et al. 2017b), namely a declining responsiveness of firms' to investment opportunities (q values) over time. This means that estimated q regression coefficients – the slope of the investment demand curve – are declining over time. We interpret declining responsiveness to investment opportunities as reflecting firms' increased market power. We show empirically that this interpretation is valid as firms' with greater market power (proxied by greater R&D spend, product market share, or profitability) tend to have a smaller (flatter) q coefficient. This supports the existing literature's findings on the role of declining competition in the economy (Philippon 2019), but shows that it is not necessarily U.S. specific and operates indirectly, through depressing the slope of the investment demand curve.

At the macroeconomic level we find a close association between the secular slowdown in estimated baseline investment rates (time-varying intercept coefficients) and firms' releasing funds externally to

[48] While at higher levels of measurement error its credible interval becomes very large relative to the coefficient size.

shareholders, creditors, and bondholders. This reflects a macroeconomic environment with an absence of good investment opportunities relative to abundant internal financing and runs counter to the view that the investment slowdown is due largely to the retention of funds to deleverage and repair balance sheets (Koo 2011), or overcome financing constraints arising from intangible asset holdings (Falato et al. 2013).

Future research might profit from interrogating further why relative and absolute investment opportunities facing the firm are declining. A growing body of models and empirical research links fiscal policy in mature economies (with a focus on 'fiscal consolidation') to secular stagnation (DeLong et al. 2012; Ollivaud et al. 2018; Fatás et al. 2018; Rachel et al. 2019; Skott 2019). Our paper makes no attempt to explore this linkage, perhaps via changing pre- and post-tax rates of return on fixed capital vs. financial capital over time. But future research might profit from doing so. Lastly, a growing body of evidence shows that increasing inequality may constrain demand growth, including fixed capital investment spending (Cynamon et al. 2015; Dabla-Norris et al. 2015; Saez et al. 2016; Auclert et al. 2018).[49] Further work is required to link this type of data to changing firm-level investment rate patterns though.

Lastly, the proper conduct of monetary policy in an environment of weak investment demand, high profitability, and stagnant investment opportunities is important, since we show that non-financial corporate money demand becomes intimately tied to the releasing of funds externally. It makes sense then that quantitative easing (QE) monetary policy (Martin et al. 2012; Lyonnet et al. 2012) would further support this behaviour, if investment opportunities remain constant.

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Appendices

Appendix (for publication with paper)

A Bayesian Hierarchical Model

A.1 Technical Model Specification

Our hierarchical model, a 'mixed, fixed and random coefficient model (Greene 2003; Hsiao 2014), are a natural extension of analysis of variance (ANOVA) models (Malinvaud 1980; Gelman 2006). Equation 11 in our main accompanying paper can formally be written in a hierarchical form as:

$$\log(y_i) \sim \mathrm{t}_\nu(\mu, \sigma_y^2, \nu_y), \tag{14}$$

$$\mu_{[i]} = X_i^0 \beta^0 + X_i \beta_{t,c[i]} + \rho \epsilon_{i,t-1}, \qquad \text{for } i \in 1:n \tag{15}$$

$$\beta_{t,c} \sim \mathrm{MVN}(M_\beta, \Sigma_{t,c}^\beta), \qquad \text{for } t,c \in 1:T,C \tag{16}$$

Equation 14 shows that our regression model is specified in log-level form. By making our dependant variable roughly normal, this dramatically improves sampling efficiency and reduces heteroskedasticity.[50] We use a symmetric student-t distribution t_ν, with the degree of freedom ν, as our likelihood function.[51] The mean of the investment function (eq.15) is the location parameter μ of the t-likelihood, and estimated as the combination of the fixed effect and random effect coefficients. X_i^0 are the fixed effect predictors, with parameter estimates β^0 from the pooled, population-level regression. X_i are the 3 random, group-level, predictors with parameter estimates $\beta_{t,c[i]}$, varying for each 'cluster' within each group of countries and years (and country:years). The time- and country-level group regressions contain 27 and 21 clusters, respectively, such that $T = 27$ and $C = 21$.[52] For each of the two groups (t, c), $\beta_{t,c}$ is a vector of length 3 random effects corresponding to the t^{th} or c^{th} row of β. Finally, $\epsilon_{i,t-1}$ is the error term at time $t-1$, where ρ represents the estimated AR(1) error process. This estimates the degree of auto-correlation in the error term, and, therefore, the state-dependence of the investment

[50] This can be seen by running simple quantile investment regressions of $\log(q)$ on investment, and plotting the fits across quantiles (Koenker et al. 2001).

[51] Although the student-t distribution becomes 'normal' shaped as $\nu_y \to \infty$, its longer tails allow it to accommodate outlying observations. A 't-likelihood' also effectively adjusts for a particular model of heteroskedastic normal errors (Arnold 2019).

[52] This structure implies that firms are 'cross-classified', with each firm belonging to only a single country, but to more than one year.

rate over time.[53]

For each group t, c, eq. 16 estimates the 3 random effects of our model $\beta_{t,c}$, as deviations around $M_\beta = \{\mu_\alpha, \mu_q, \mu_{cf}\}$, the grand mean of each of our 3 random effect predictors, drawn from a common multivariate normal (MVN) distribution.[54] The variance-covariance matrix Σ_β, is estimated separately for each t, c group of random effect parameters, with the 3 variance parameters in each group (one for each random effect $\sigma_\alpha, \sigma_q, \sigma_{cf}$), determining the extent of variability in parameter estimates across country or year clusters within each group.

As the key quantities of interest of our investment model are *cash flow*, q (Market-to-book or MTB ratio), and the *intercept*. They are estimated as both *fixed effects* and *random effects*, as recommended by Schmidt-Catran et al. (2015), among others. They are included in every level of our model and are the only predictors for the country and year groups. In our 'fixed' population regression level, we also include a firm size dummy and an industry dummy.

For computational purposes, the actual model is implemented and estimated using standardized group-level effects. Under this specification, our population means μ_α enter the population regression, leaving the prior on the random effects with a mean of zero. The random effects are also transformed into z-scores, $Z_{t,c}$, giving them a fixed prior that is unit normal. As a result the estimated population-level fixed effect parameters of cash flow, q, and the intercept – $\beta^0_{cf}, \beta^0_q, \beta^0_\alpha$, respectively – would be indistinguishable from their estimated population means in the random effects distribution $\mu_{cf}, \mu_q, \mu_\alpha$, respectively. As a result, $X^0_i \beta^0$ only contains the fixed effects that have no random effect counterpart. For details see Betancourt et al. (2015).

A.2 Hierarchical Priors and Variance-Covariance Structure

Below we write our variance-covariance structure more explicitly, beginning with the random effects being drawn from a wider population distribution, governed by hyper-parameters $(M_\beta, \Sigma^\beta_{t,c})$:

[53] For computational reasons, we do not apply the error structure to the covariance matrix. This is also why we do not use a higher order AR process, since model improvement, judged by Bayesian R^2, is minimal while computational time increases considerably. Also, note that this auto-correlation structure is not independent from the random effects components, even though they are defined in separate parts of the model specification. This is because the fixed effects, random effects, and auto-correlation components all go into the same regression for Y, and so are estimated together.

[54] Later we use group predictors to model $\mu_\alpha = \gamma^\alpha_0 + \gamma^\alpha_1 \mu$, where μ will vary for each group $\{t, c\}$. X_i matrix is, therefore, able to contain group-level predictors too.

$$\begin{pmatrix} \alpha_{t,c} \\ \\ \beta^{q}_{t,c} \\ \\ \beta^{cf}_{t,c} \end{pmatrix} \sim \text{MVNormal} \left[\begin{pmatrix} \mu_{\alpha} \\ \\ \mu_{q} \\ \\ \mu_{cf} \end{pmatrix}, \quad \Sigma^{\beta}_{t,c}, \right], \tag{17}$$

Each group t, c has its own variance-covariance matrix (though we do not write it out twice). Within each group, the variance-covariance matrix (eq. 18) is $\Sigma^{\beta} = \mathrm{D}(\sigma)\,\Omega\,\mathrm{D}(\sigma)$, where $\mathrm{D}(\cdot)$ has the standard deviation of each of the 3 random effect variables along the diagonal:

$$\Sigma^{\beta}_{t,c} = \begin{pmatrix} \sigma_{\alpha_{t,c}} & 0 & 0 \\ \\ 0 & \sigma_{\beta^{q}_{t,c}} & 0 \\ \\ 0 & 0 & \sigma_{\beta^{cf}_{t,c}} \end{pmatrix} \Omega \begin{pmatrix} \sigma_{\alpha_{t,c}} & 0 & 0 \\ \\ 0 & \sigma_{\beta^{q}_{t,c}} & 0 \\ \\ 0 & 0 & \sigma_{\beta^{cf}_{t,c}} \end{pmatrix}. \tag{18}$$

Ω shows the correlation between the random effect coefficients for different variables, such that we have:

$$\Omega_{t,c} = \begin{pmatrix} 1 & \rho_{\alpha_{t,c},\beta^{q}_{t,c}} & \rho_{\alpha_{t,c},\beta^{cf}_{t,c}} \\ \\ \rho_{\alpha_{t,c},\beta^{q}_{t,c}} & 1 & \rho_{\beta^{q}_{t,c},\beta^{cf}_{t,c}} \\ \\ \rho_{\alpha_{t,c},\beta^{cf}_{t,c}} & \rho_{\beta^{q}_{t,c},\beta^{cf}_{t,c}} & 1 \end{pmatrix}. \tag{19}$$

We put a standard LKJ prior on the covariance matrix of the multivariate *normal* distribution, with $\eta = 1$, such that the prior independence between coefficients — a diagonal co-variance matrix — is the default. The full list of priors is the following:

$$\beta^{0}_{\alpha} \sim \text{Student-t}(3, -3.1, 2.5), \tag{20}$$

$$\nu \sim \text{Gamma}(2, 0.1), \tag{21}$$

$$\sigma_{y},\ \sigma_{\alpha,q,cf \in t},\ \sigma_{\alpha,q,cf \in c},\ \sigma_{\alpha,q,cf \in j} \sim \text{Student-t}^{+}(3, 0, 2.5), \tag{22}$$

$$\mathbf{R} \sim \text{LKJcorr}(1)\,. \tag{23}$$

where Student-t^{+} represents a half Student-t prior defined only on the positive support. β^{0}_{α} is the grand intercept so we use the median value of the log investment rate for the location parameter of the Student-t prior. For the location parameter of the student-t prior on the grand intercept, we use the median value of log investment rate in the entire sample. We use non-informative prior (flat prior) for all other parameters not mentioned above.

On the LKJ prior: The multivariate normal density and LKJ prior on correlation matrices both require their matrix parameters to be factored. This is achieved by parameterizing the model directly in terms of Cholesky factors of correlation matrices using the multivariate version of the non-centered parameterization. The Cholesky decomposition is: $\Sigma^{\beta} = \mathrm{LL}^{\mathbf{T}}$, where L is a lower-triangular matrix. Inverting Σ^{β} is numerically unstable and inefficient. This is the preferred modern Bayesian prior (Stan Development Team 2019). The LKJ distribution for correlation matrices is $\mathrm{LKJcorr}(\Omega|\eta) \propto \det(\Omega)^{\eta-1}$, where $\eta > 0$ determines the degree of correlations (Lewandowski et al. 2009). The LKJ distribution behaves similarly to the beta distribution for scalars. $\eta = 1$ is a special form of a non-informative uniform distribution on correlation, $\eta > 1$ leads to less correlation between group-level coefficients, with more mass concentrated around the identity matrix, while $\eta < 1$ leads to stronger prior correlation between group-level coefficients as more mass is concentrated in the other directions. We use LKJ prior with $\eta = 1$ as the default. For robustness we run the models with $\eta = 5$, and the results are essentially the same.

Lastly, using Bayes rule, we present a very general form of the posterior density of our unknown parameters conditional on the data. Given the student-t likelihood and the multivariate normal prior, we have the following joint posterior parameter distribution, with N number of observations, K number of predictors and, L number of groups:

$$\begin{aligned} p(\theta|y) &\propto p(y|\theta)\, p(\theta|\phi)\, \mathrm{p}(\phi) \\ &\propto \underbrace{\prod_{l=1}^{L} \text{student-t}(y_{.l}|\beta_l, \nu, \sigma_y)}_{\text{Likelihood}} \underbrace{\prod_{l=1}^{L} \mathrm{MVN}(\beta_l|\boldsymbol{M_\beta}, \boldsymbol{\Sigma^\beta})}_{\text{Prior}} \underbrace{p(\boldsymbol{M_\beta}, \boldsymbol{\Sigma^\beta})}_{\text{Hyper prior}} \end{aligned} \tag{24}$$

where y, θ, and ϕ denote the data, parameters of the likelihood function, ϕ is the parameters of the prior distribution on group-varying components of θ. Since $p(\boldsymbol{M_\beta}, \boldsymbol{\Sigma^\beta})$ is the prior distribution on the parameters of the prior distribution, we call this a *hyper prior* distribution.

A.3 Variational Inference

Given how complex our posterior joint distribution is, variational inference can reduce computational time by several days through optimizing a related posterior distribution, while approximating the true posterior very well - and in many respects with greater accuracy. Variational inference (VI) uses optimization to find a simpler, more tractable, distribution $q(\mathbf{Z})$, from a family of distributions $\mathcal{Q}$, that it is close to the desired posterior distribution $p(\mathbf{Z} \mid \mathbf{X})$. In VI, "close to" is defined using the Kullback-Leibler (KL) divergence. The desired VI objective is (Blei et al. 2017):

$$q^{\star}(\mathbf{Z}) = \arg\min_{q(\mathbf{Z}) \in \mathcal{Q}} D_{\mathrm{KL}}[q(\mathbf{Z}) \| p(\mathbf{Z} \mid \mathbf{X})].$$

The goal is to find an optimal distribution $q^{\star}(\mathbf{Z})$, where optimality is defined as having the smallest possible KL divergence between $q^{\star}(\mathbf{Z})$ and $p(\mathbf{Z} \mid \mathbf{X})$. The absolute minimum of this divergence is zero. Minimizing the KL divergence can be interpreted as minimizing the relative entropy between the two distributions. In VI, however, the desired KL divergence cannot be computed, so instead one optimizes a different objective that is equivalent to this KL divergence up to constant (ibid.). This new objective is called the evidence lower bound (ELBO). So by maximizing the ELBO, we minimize the desired KL divergence. The iterative algorithm which we use for the variational inference is the so-called 'mean field' approximation. If the constrained space $\mathcal{CC}$ is confined within independent space, i.e. $q^{\star}(\mathbf{Z}_1 \mid \mathbf{Z}_2) = q^{\star}(\mathbf{Z_1})$, the iterative scheme will become the so-called mean field approximation $Q^{\star}(\mathbf{Z}) = q^{\star}(\mathbf{Z}_1)q^{\star}(\mathbf{Z}_2)$,.

A.4 Model Fit

Table 4. Model Fit: Bayesian R^2 by Country and Year Groups

Year	R2	Est.Error	Q2.5	Q97.5
1994	0.06	0.01	0.04	0.09
1995	0.07	0.02	0.04	0.10
1996	0.07	0.02	0.04	0.10
1997	0.07	0.02	0.04	0.10
1998	0.07	0.02	0.04	0.10
1999	0.08	0.02	0.04	0.12
2000	0.07	0.02	0.04	0.11
2001	0.06	0.02	0.03	0.10
2002	0.06	0.01	0.03	0.08
2003	0.06	0.01	0.03	0.09
2004	0.06	0.01	0.03	0.09
2005	0.06	0.01	0.04	0.09
2006	0.07	0.01	0.04	0.10
2007	0.07	0.01	0.04	0.10
2008	0.06	0.01	0.04	0.09
2009	0.05	0.01	0.03	0.08
2010	0.06	0.01	0.04	0.09
2011	0.06	0.01	0.04	0.09
2012	0.06	0.01	0.04	0.09
2013	0.06	0.01	0.04	0.09
2014	0.06	0.01	0.04	0.09
2015	0.06	0.01	0.04	0.08
2016	0.06	0.01	0.04	0.08
2017	0.05	0.01	0.03	0.08
2018	0.05	0.01	0.04	0.08
2019	0.05	0.01	0.03	0.07
2020	0.05	0.01	0.03	0.07

Country	R2	Est.Error	Q2.5	Q97.5
AUS	0.25	0.01	0.23	0.27
BMU	0.26	0.01	0.24	0.28
CAN	0.25	0.01	0.23	0.27
CHE	0.25	0.01	0.23	0.27
CYM	0.25	0.01	0.23	0.27
DEU	0.25	0.01	0.23	0.27
DNK	0.25	0.01	0.23	0.27
ESP	0.25	0.01	0.23	0.27
FIN	0.26	0.01	0.24	0.28
FRA	0.26	0.01	0.23	0.27
GBR	0.26	0.01	0.23	0.27
HKG	0.26	0.01	0.23	0.27
ISR	0.25	0.01	0.23	0.27
ITA	0.26	0.01	0.23	0.28
JPN	0.26	0.01	0.24	0.28
NLD	0.26	0.01	0.24	0.28
NOR	0.25	0.01	0.23	0.27
NZL	0.26	0.01	0.24	0.28
SGP	0.25	0.01	0.23	0.27
SWE	0.25	0.01	0.23	0.27
USA	0.25	0.01	0.23	0.27

Note: *The mean (R^2), Standard deviation (Est.Error) and the 95% credible interval are reported for each Bayes R^2. Note that R^2 for the year-level prediction is substantially lower than for the country-level.*

B Robustness: Measurement Error Model

Attenuation bias is a common concern in investment regression specifications and has shown to be significant: materially impacting the size and significance of cash flow coefficients (downwards) and q coefficients (upwards) (Erickson and Whited 2000).

We apply a Bayesian measurement error correction to both the fixed effect and the random effects of observed q. To our knowledge this is the first time a Bayesian error correction model has been applied to a cash flow-q regression. This has the impact of increasing the size of the q coefficients - both the fixed effects and the random effects - in non-linear proportion to the assumed degree of attenuation.[55]

A Bayesian approach to measurement error is computationally demanding but has several advantages. Firstly, the Bayesian estimator provides a posterior distribution that takes into account uncertainty due to estimating other parameters. In contrast, the classical estimator corrected for attenuation would require bootstrapping or some type of asymptotic approximation to account for this uncertainty. Secondly, Bayesian inference averages over plausible values of mismeasured Q in light of the data, rather than imputing a single best-guess and then proceeding as if this guess is correct. Uncertainty in estimation of Q is then propagated forward. Thirdly, we can integrate the measurement error with a more complex model: largely keeping our random effects structure, an autoregressive error structure, a student-t likelihood, and other deviations from a simplistic panel regression model (Carroll et al. 2006).

A Bayesian approach to measurement error is formulated by treating the true quantities being measured as missing data (Clayton 1992; Richardson et al. 1993; Gelman, Carlin, et al. 2013). This requires a model of how the measurements are derived from the true values. In what follows Q is an imperfectly measured surrogate for the unobservable $\widetilde{Q}$ measured without error. We assume classical measurement error such that $Q = \widetilde{Q} + \epsilon$. This implies greater variability in the observed surrogate, Q, than true $\widetilde{Q}$. The error is assumed to be homoskedastic with zero mean and identity covariance matrix independent of true covariates, $\text{Var}(\epsilon|\widetilde{Q}) = \tau_{me}\boldsymbol{I}$, where τ_{me} governs the variance of the measurement error ϵ. This implies that surrogate Q is an unbiased version of the true covariate $\widetilde{Q}$, hence $E(Q) = E(\widetilde{Q})$.

We assume a normal model for our error term as well as multiplicative measurement error to begin with, such that $Q = \widetilde{Q}\epsilon$ (Iturria et al. 1999), which with our log-log *investment-q* model then turns into an additive error model $log(Q) = log(\widetilde{Q}) + \epsilon$. As such our measurement error model is additive.

[55] This is called a ‘sensitivity analysis’.

This leads to the following measurement error model on the fixed effect and random effect Q values:[56]

$$Q_{ij} \sim \mathcal{N}(\widetilde{Q}_{ij}, \tau_{me}), \tag{25}$$

$$Q_i \sim \mathcal{N}(\widetilde{Q}_i, \tau_{me}). \tag{26}$$

Adding in a measurement error model for q introduces the additional unknown $\tilde{q}$, with a joint posterior $h(y, q, \tilde{q}, z)$. Given our mixed effects multilevel model this integral cannot be solved directly as it is too complex. But Bayesian methods can be used to sample from the distribution. We make the following assumption when factoring the above joint distribution: Y and Q^* are conditionally independent given true covariates $\{Z, X\}$. This is the *nondifferential measurement error* assumption: $h(y|q, \tilde{q}, z) = h(y|q, z)$. With this assumption we have:

$$\begin{aligned} h(y, q, \tilde{q}, z) &= h(y|q, \tilde{q}, z)\ h(q, \tilde{q}, z) && (27) \\ &= h(y|q, z)\ h(q, \tilde{q}, z) && (28) \\ &= h(y|q, z)\ h(\tilde{q}|q, z)\ h(q, z). && (29) \end{aligned}$$

We do not adopt a so-called 'structural modelling' common to likelihood based measurement error methods, which involves elaborating the joint density of the true covariates into an 'exposure model' of the type $h(q, z) = h(q|z)h(z)$. We have no specific interest in the distribution of the precisely measured covariates $h(z)$, and so dispense with a model for them. Instead we treat the joint distribution of the true covariates as fixed (so-called 'functional method') - thereby basing inferences conditioning on $\{Q, Z\}$. This has the benefit of being robust to distributional assumptions regarding $h(q)$ and computationally more efficient, but at the cost of not modelling any explicit dependence between q and z. As a result, we model the conditional distribution of the outcome variable given the observed covariate variables as (Grace 2016):

$$f(y|\tilde{q}, z; \theta) \propto \int\ f(y|q, z; \beta)\ f(q|\tilde{q}, z)\ d\eta(q). \tag{30}$$

[56] We treat τ as data rather than as a parameter. As a result no prior is put on τ. This increases computation speed and facilitates identifiability for the measurement error model, but comes at the cost of reducing the uncertainty in our parameter estimates. We do, however, put a prior on $\widetilde{Q}$. The uncertainty of the measurement error model will partially be reflected in the estimate of the population parameters of perfectly measured $\widetilde{Q}$, and in particular in σ_Q

This leads to the following model:

$$y_i \sim \mathrm{t}_{\nu}\left(X^0_{i-Q}\beta^0 + X_{i-Q}\beta_{j[i]} + \widetilde{Q}^0_i\beta^0 + \widetilde{Q}_i\beta_{j[i]},\, \sigma^2_y,\, \nu_y\right) \quad \text{for } i = 1, \ldots, n, \tag{31}$$

$$Q_{ij} \sim \mathcal{N}\left(\widetilde{Q}_{ij}, \tau_{me}\right), \tag{32}$$

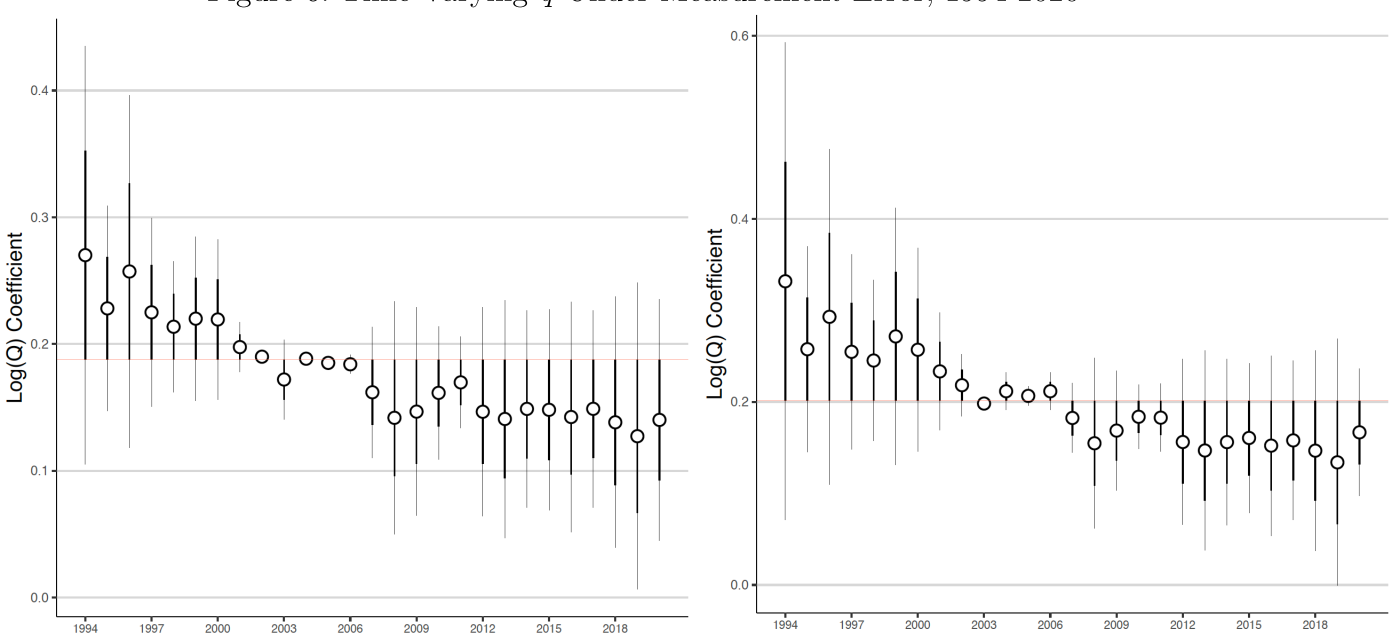
$$Q_i \sim \mathcal{N}\left(\widetilde{Q}_i, \tau_{me}\right), \tag{33}$$

$$\beta_j \sim \mathrm{MVN}\left(M_\beta, \Sigma_\beta\right) \quad \text{for } j = 1, \ldots, J. \tag{34}$$

We provide no additional ('exposure') model for true Q - which does not contribute very much to inferences generally except under certain circumstances (Fuller 1987; Gustafson 2003, pp. 85-92). Another way of thinking about the measurement error model for Q is as an additional random effects model, where measured q is drawn from a population with a true population mean and variance estimated from the data.

Our prior for $\tilde{Q}$ is centred at τ. As the value of τ increases or decreases our prior increases in turn. This is done purely for computational purposes. The results are summarised in Table 5 below for $\tau = \{0.1, 0.3, 0.5\}$. This is called a sensitivity analysis, where τ proxies, very roughly, for the assumed degree of measurement error.[57]

Figure 6. Time-Varying q Under Measurement Error, 1994-2020



Note: *Time-varying Q trend remains downward sloping under measurement error, with a moderation in trend from 2011. Credible intervals increase for most years.* $\tau = 0.3$ *on the left hand side and* $\tau = 0.5$ *on the right hand side.*

[57] In a single variate case, $\tau = 0.5$ very roughly amounts to 50% measurement error in q, and 20% relative bias in q, defined as $(Q - \widetilde{Q})/\widetilde{Q}$. But under high correlation between cash flow and q this relationship can be greatly magnified. For this reason alone τ of 0.5 or above can be considered to amount to large measurement error in q. Moreover, since the sample variance of $\log(q)$ is 0.69 this implies a very large error indeed at $\tau = 0.5$ (which is the variance of the error distribution around true q) (Gustafson 2003).[58]

Table 5. Sensitivity Analysis of Hierarchical Model with a Single Group to Differing Degrees of Attenuation Bias

		Non ME		**ME .1**		**ME .3**		**ME .5**	
	Variable	Est.	Est.Err.	Est.	Est.Err.	Est.	Est.Err.	Est.	Est.Err.
Fixed Effect	α	-3.11	0.01	-3.11	0.01	-3.02	0.01	-3.07	0.01
	β^{cf}	-0.01	0.00	-0.01	0.00	-0.05	0.00	0.02	0.01
	β^{q}	0.12	0.00	0.15	0.00	0.19	0.00	0.20	0.00
Country Random Effect	σ_{α_c}	0.15	0.00	0.17	0.00	0.17	0.00	0.17	0.00
	$\sigma_{\beta_c^{cf}}$	0.07	0.00	0.07	0.00	0.08	0.00	0.06	0.00
	$\sigma_{\beta_c^{q}}$	0.04	0.00	0.05	0.00	0.04	0.00	0.05	0.00
	$\rho_{\alpha_c,\beta_c^{cf}}$	-0.02	0.06	-0.04	0.06	-0.02	0.05	0.01	0.09
	$\rho_{\alpha_c,\beta_c^{q}}$	0.01	0.08	-0.08	0.05	0.11	0.06	0.52	0.05
	$\rho_{\beta_c^{cf},\beta_c^{q}}$	0.16	0.07	-0.09	0.06	-0.09	0.07	-0.01	0.07
Year Random Effect	σ_{α_t}	0.12	0.00	0.19	0.00	0.13	0.00	0.17	0.00
	$\sigma_{\beta_t^{cf}}$	0.05	0.00	0.06	0.00	0.05	0.00	0.05	0.00
	$\sigma_{\beta_t^{q}}$	0.02	0.00	0.02	0.00	0.02	0.00	0.04	0.00
	$\rho_{\alpha_t,\beta_t^{cf}}$	-0.32	0.03	-0.52	0.03	-0.36	0.03	-0.47	0.04
	$\rho_{\alpha_t,\beta_t^{q}}$	0.53	0.07	0.73	0.04	0.69	0.03	0.78	0.03
	$\rho_{\beta_t^{cf},\beta_t^{q}}$	-0.19	0.07	-0.44	0.06	-0.33	0.06	-0.47	0.05
AR(1) Parameter	φ	0.74	0.00	0.71	0.00	0.72	0.00	0.73	0.00
Student-t Parameters	σ	0.54	0.00	0.52	0.00	0.53	0.00	0.52	0.00
	ν	2.52	0.02	2.56	0.01	2.53	0.01	2.54	0.01

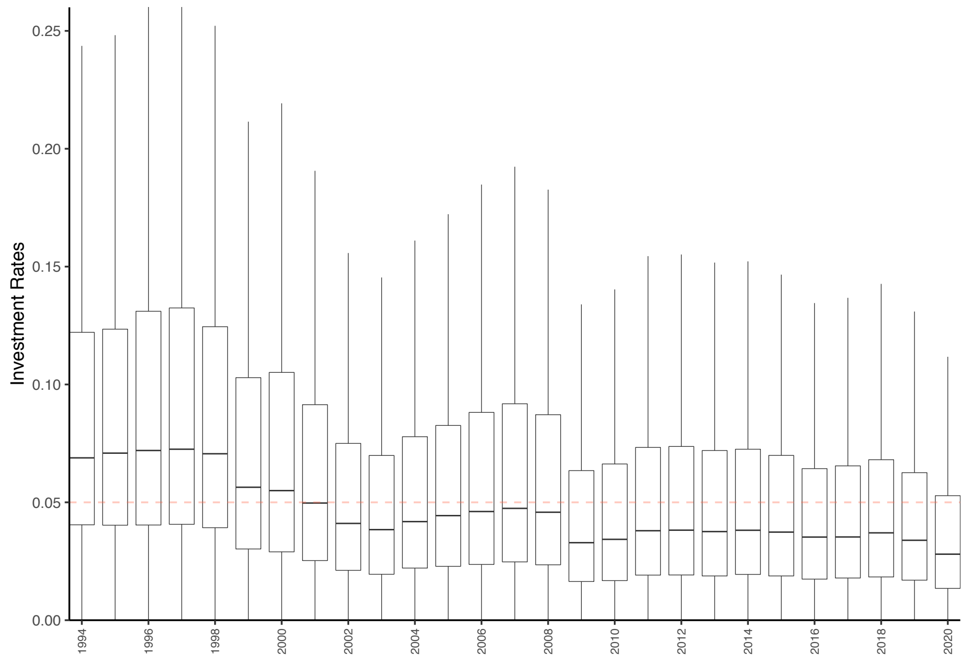
Note: *Comparison of posterior estimates for baseline mixed hierarchical model and measurement error models applied to q. Three different values of τ are tested. Standard deviation coefficient, the mean (Est.) and the standard deviation (Est.Err) are reported. As τ increases the size of the fixed effect and the standard deviation of the random effect q coefficients increases non-linearly.*

The size of the fixed effect value of q, β^q, increases as the value of τ increases, from 0.12 in the baseline model, to 0.15 ($\tau = 0.1$) and 0.19 ($\tau = 0.3$). In terms of the cashflow, its fixed effect coefficient value β^{cf} declines gradually relative to the non-measurement error model: from -0.01 (baseline model) to -0.05 ($\tau = 0.3$). At $\tau = 0.5$ the coefficient becomes positive but the posterior credible interval explodes with the estimated error increasing to 0.01. This finding is as one would expect from a measurement error model with non-zero correlation between these regressors (Gustafson 2003).

In terms of time-varying Q: the key downward trend remain largely the same, with some softening in the decline from 2011. This is shown in Figure 6, with $\tau = 0.3$ on the left hand side and $\tau = 0.5$ on the right hand side.

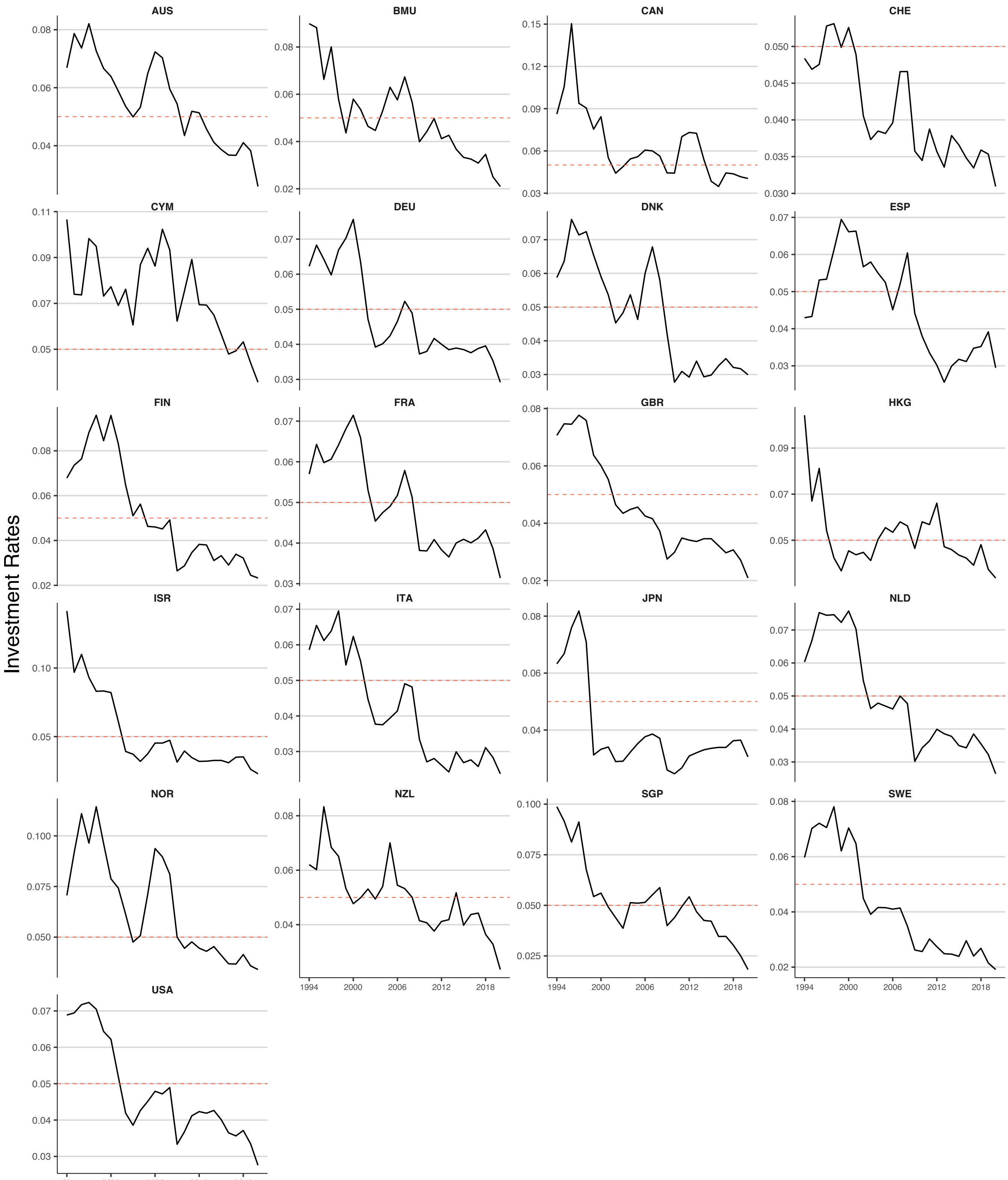
C Movement of Key Variables by Time and Country Group

Figure 7. Pooled Developed Economy Firm-Level Investment Rates, 1994-2020



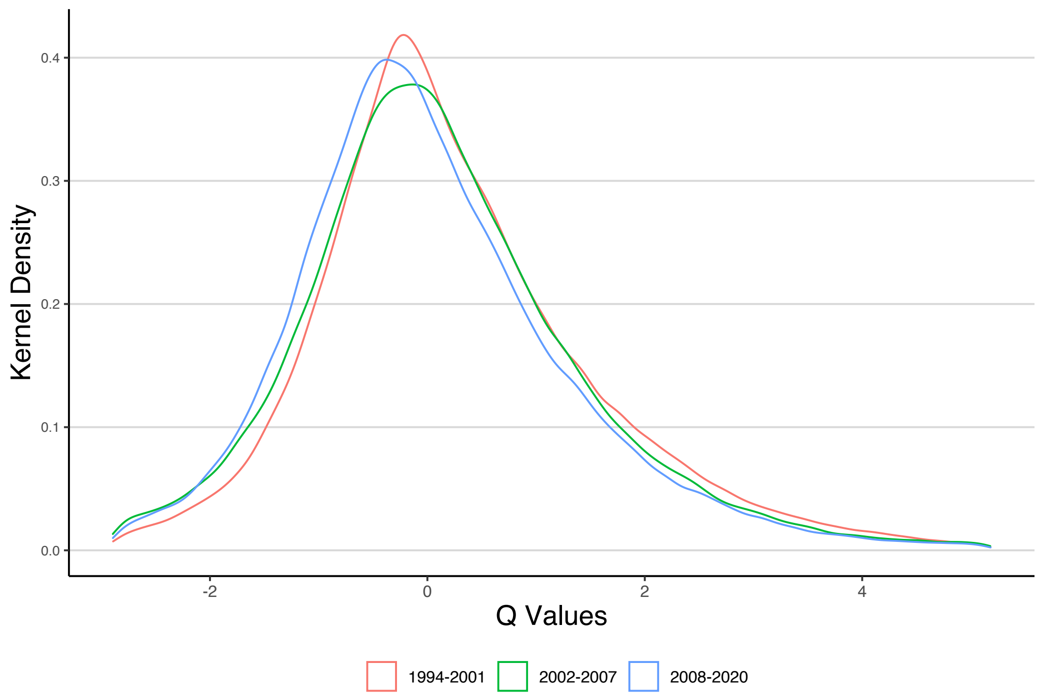
Note: *Showing box plots of firm-level investment rates with horizontal red line at 5% investment rate. After 1998 and 2008 we see a structural drop. Year median is bold horizontal lines within each box plot. Sample consists of firms incorporated in Australia, Bermuda, Canada, Switzerland, Cayman Islands, Germany, Denmark, Spain, Finland, France, Great Britain, Hong Kong, Israel, Italy, Japan, Netherlands, Norway, New Zealand, Singapore, Sweden, and USA. 'Outliers' (observations outside of 1.5 IQR) not shown.*

Figure 8. Median Investment Rate by Advanced Economy, 1994-2020



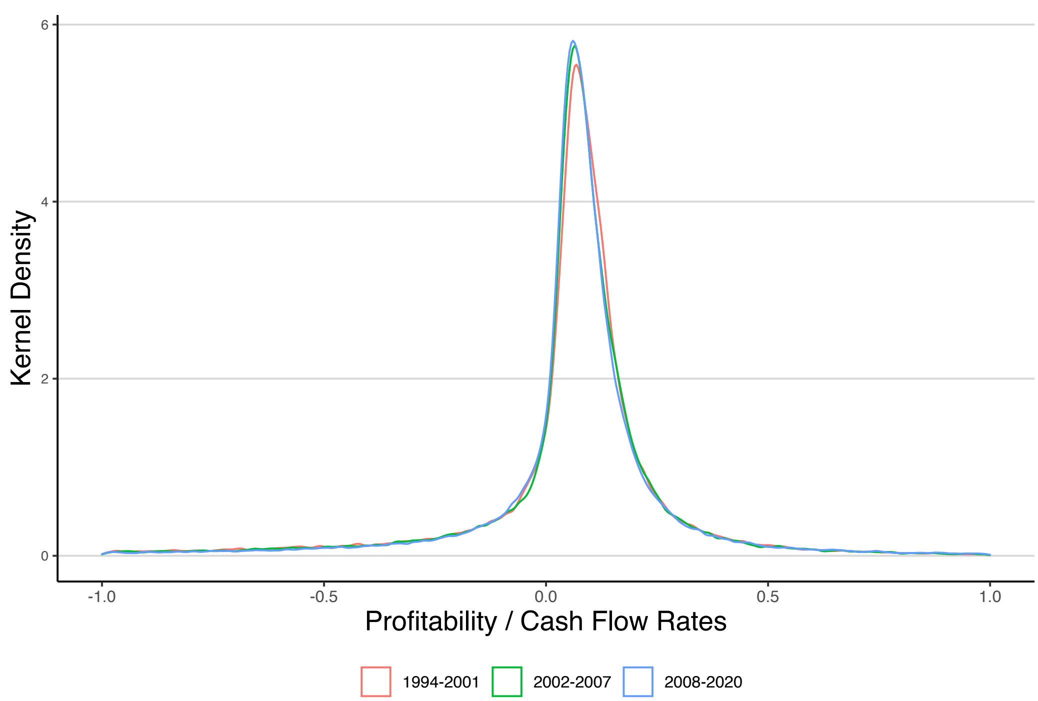
Note: *Median investment rate by country with horizontal reference line at 5%. Showing declines across advanced economies. Sample size increases from 1999 for most countries. Japan shows different trend.*

Figure 9. Density of Q Values (Log2 scale), by time period



Note: *Q values (investment opportunities) shift in over time (to the left) declining. The mode shows a large shift in the time group 2008-2020. Prior to this the distribution's tails are moving to the left while the mode remains constant.*

Figure 10. Density of Cash Flow Rate, by time period



Note: *Cash flow rate is largely very stable over time. After the first time period the mode shifts to the left but the tails show some movement. Removing more extreme values (limits at ±100%).*

Online Appendix

A. Data and Variables

Our sample combines S&P Compustat North America and Compustat Global databases, downloaded from the WRDS interface. Combining these two datasets is done through use of the Compustat exchange rate file to convert into a common currency (US$). Compustat North America follows GAAP accounting standards while the rest of the world largely follows IFRS to varying extents and in different forms. Familiarity with these accounting models can help one understand differences and similarities in variables (for example PWC 2018). Our variables are reported gross, i.e. before amortization and depreciation, but after tax, unless stated otherwise. All dates and plots are for the fiscal year rather than the calendar year.

C.1 Data Cleaning

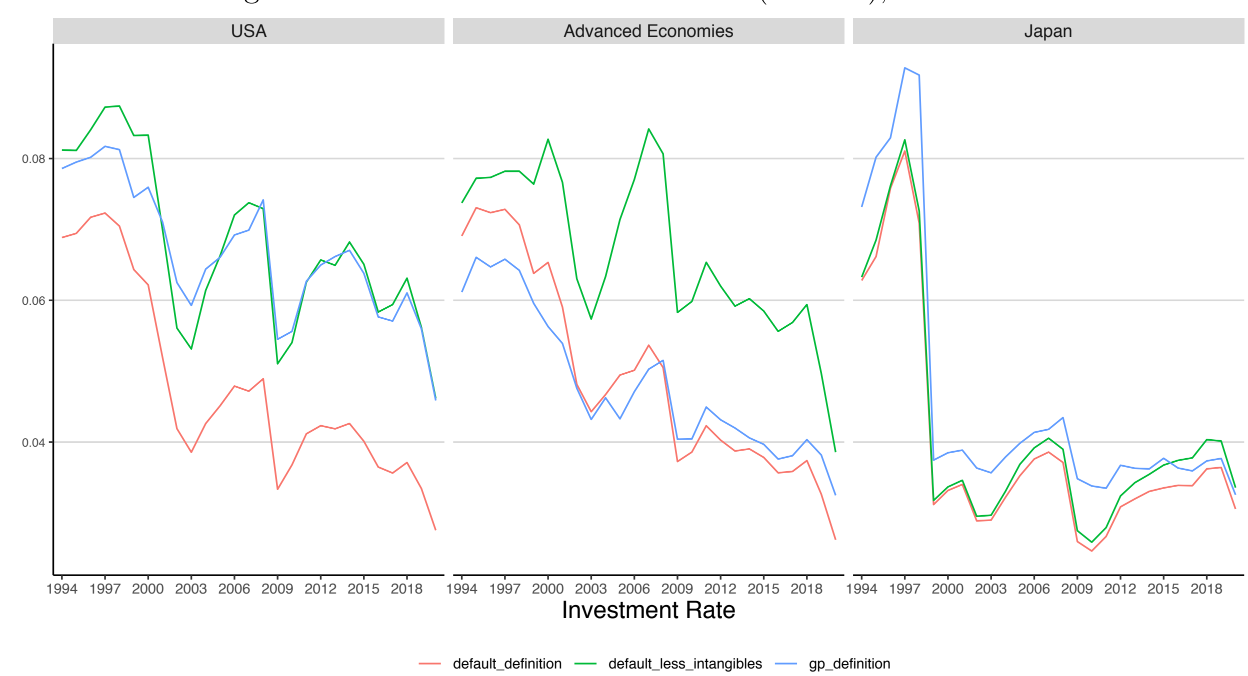
Data cleaning is done on the combined sample of Compustat North America and Compustat Global, after which we select our advanced economy sub-sample, based on GDP per capita and meeting our minimum number of observations cut-off. *The first round of data processing* limits the dataset to firms with positive values for all four of the following: gross capital stock (property, plant, and equipment), capital expenditure, sales, and NA values for the gsector variable. We exclude firms working in gardens, zoos, museums, non profit organisations, and utilities, but keep gas production and distribution. We remove financial companies but keep real estate and certain other related companies. This amounts to removing SIC codes 84, 86, 493-499, 60-64, and 66-69. *The second round of data processing:* We replace NA values with zero values for Goodwill, Intangible assets, Inventories, and R&D spending (though we do not use R&D in this paper). For intangibles this approach follows Peters et al. (2017). A few dozen companies report intangibles incorrectly, such that Goodwill is larger than total intangible assets, of which it is a subset. We correct this to ensure that intangibles is at least as big as Goodwill. All variables are created at this stage (except firm size dummy). Any infinite values are removed after dividing by capital stock. *The third round of data processing:* We trim (i.e. remove) the top and bottom 0.05% of observations by profit rate and investment rate. We trim the top and bottom 1.5%. We make the size variable once this trimming has taken place.

C.2 Variable Definitions

Capital Stock: Is defined gross (i.e. before depreciation and amortisation) as the sum of gross property, plant and equipment (PPEGT in Compustat), intangible assets INTAN, and inventories INVT. Our preferred capital stock measure includes intangibles and inventories, though our findings are not dependant on them. The BEA measure of capital stock now includes intangible assets (including software, R&D, and some intellectual property). Studies tend to include intangibles in their capital stock measure or at least adjust for it now (Fernald et al. 2017; Peters et al. 2017). See also: Haskel et al. (2018). However, intangible assets are measured net. Various simple methods of adjustment can be undertaken but did not appear to materially impact the results. More complex adjustment can be found in Peters et al. (2017), who notes a positive impact on q coefficient values from the inclusion of intangible assets. Gross investment rates are recommended rather than 'net' for cross-country comparisons for national accounts and firm-level data (Lequiller et al. 2014). GAAP and IFRS contain important differences in depreciation rules, implied by how development costs are capitalized differently, and also differences in how impairment losses and component depreciation are treated.

We plot three definitions of investment rates below (Figure 11), showing the median value.

Figure 11. Investment Rate Definitions (median), 1994-2020



Note: *Showing median investment rates. "gp" is the Gutiérrez et al. (2017b) definition of the investment rate as* $(capx+R\&D)/AT$, *"default" is our definition of investment rates as* $(CAPX + INVT + INTAN)/PPEGT$. *While "default less intangibles" is the same less* $INTAN$.

Inclusion of intangible assets has the effect of lowering investment rates during the mergers &

acquisitions boom in the 2000s. In general it leads to less volatility in the investment series for almost all countries. R&D relative to assets is much higher in the U.S. than other advanced economies, such that its inclusion sees investment rates increase in the U.S. by far more than in our advanced economies pool. The definition used by Gutiérrez et al. (2017b) has no major impact at the median for advanced economy firms as a whole, but has a clear impact for the U.S. – showing a much more volatile cycle, though still with a clear trend.

'Rates': Profit rate and investment rate (and all 'rate' variables) are defined over the firms (gross) capital stock as the denominator.

Profit Rate vs. Cash Flow Rate: We use profits instead of cash flow as our preferred indicator of finance constraint / availability of internal finance. Profits and cash flow are very similar, both measured in this case gross after taxes and interest payments. Operating cash flow (OANCF) is measured gross (after taxes and interest payments) but after making adjustments for changes in working capital and other non-operating income to account for all cash going in and out of the firm (including from sales of assets, and changes in inventory and accounts receivable. Cash flow rates on fixed capital will, however, be somewhat exaggerated in our opinion since OANCF includes dividends received by the firm, but does not deduct dividends made, as is standard accounting practise. Our measure of profits (operating income before depreciation and amortization) is also after making deductions for taxes paid and interest payments but includes only revenue generated through sales.

Profit: We define profit from the income statement as OIBDP - TXT - XINT or gross operating income before depreciation and amortization after deducting taxes, and interest payments and related expenses.

FINCF: Is net cash flow from financing activities, and includes all activities to release or inject finance into the firm. We normalize this variable by sales.

Firm Size: The firm size dummy is a rough proxy and consists of 10 equal bin dummies based on the firm's sales. We bin the variable using the *cut2()* function in R. This ensures that an equal number of observations are in each bin unless this would not be ideal for the optimisation algorithm. The mean value in each bin is used as the bin label.

Industry Classification - GICS: We use Compustat's gsector variable which is based on the Global Industry Classification Standard (GICS). This is an industry taxonomy developed in 1999 by MSCI and S&P for use by the global financial community. The GICS structure consists of 11 sectors, 24 industry groups, 69 industries and 158 sub-industries into which S&P has categorized all major public companies. The system is similar to ICB (Industry Classification Benchmark), a classification structure maintained by FTSE Group. It is based on the company's principal business activity. As a new classification system it accounts well for technology firms.[59] As a result, in the U.S., the SIC code has been replaced by the North American Industry Classification System (NAICS code), which was released in 1997. Some U.S. government departments and agencies, such as the U.S. Securities and Exchange Commission (SEC), continued to use SIC codes through at least 2019.

Tobin's *q*: We calculate Tobin's q as the firm's market-to-book ratio (MTB). Book values (the denominator) is calculated the same across all countries in our sample. Book value is equal to total assets (AT). Market value calculations differ, however, between Compustat Global and Compustat North America. *For Compustat North America* market value of the firm is easy to calculate and equal to the market capitalization of the firm's equity plus the book value of the firms debt: CSHO * PRCC_F + DLC + DLTT, while the book value of assets is AT. There is no need to adjust (i.e. multiply) CSHO by AJEX, which accounts for stock splits and stock dividends, since this factor cancels out.

For Compustat Global the process of calculating the 'equity market capitalization' component is somewhat more involved and requires making additional assumptions. Data is downloaded for the last available month of the year ('end of month' filter) and when 'earnings participation flag' is equal to 'yes'. The company may have market values on several exchanges globally. Market capitalization is calculated across each exchange before being aggregated across, whereby we have QCSHOC = ((CSHOC*QUNIT)/1,000,000), marketcap = PRCCD*QCSHOC and marketcap_T =sum(marketcap), across all exchanges, where shares outstanding are CSHOC, and PQUNIT represents the size of the block in which the shares are quoted on the exchange. In particular see Compustat (2009) for further details. As with Compustat North America our calculation excludes non-traded shares. PRCCD Is the daily securities price and should be multiplied by AJEXDI to acccount for stock splits.

The literature tends to define q as Market Value of Fixed Capital / Book Value of Capital. Erickson and Whited (2006) finds this performs only marginally better than other measures, such as market-to-book value of the firm, in explaining investment rates. We instead use the firm's market-to-book ratio (MTB) of total assets as our proxy for Tobin's q. MTB captures average rather than margin q

[59]By comparison, SIC industry codes are difficult to use beyond the one digit level as they disaggregate too quickly and in categorisations which are not that informative. SIC codes were developed for traditional industries prior to 1970. Business has changed considerably since then from manufacturing to services based production. As a result, the SIC has been slow to recognize new and emerging industries, such as those in the computer, software, and information technology sectors (with these classifications often less intuitive.

which is only equal under restrictive assumptions (Hayashi 1982).

Use of MTB is motivated by several considerations, including that it keeps q strictly positive as deductions for cash and non-capital items are not explicitly made. Theoretically, the meaning of a negative Tobin's q is unclear, 'what is a negative investment opportunity?'. And practically, in Compustat Global (and North America to a lesser extent) many negative q values would exist when defined as the market-to-book value of capital only, especially for Japanese firms and after the 2008 financial crisis. Moreover, its explanatory power is roughly the same as other q measures (Erickson and Whited 2006, 2012). Damodaran (2013) notes in particular that non-traded shares, management options, non-traded debt, off-balance sheet debt, trapped cash, and convertible securities can all lead to measurement error in enterprise value which ideally one should adjust for. In particular, cross-holdings in other companies may upwardly bias the (consolidated market) value of the enterprise. A closer look at the top 4% of pooled q values in our entire sample shows that holding companies feature very strongly. This also helps explain in part why firms in the Cayman Islands and Bermuda have such large q values.

The above also implies that for cross-country purposes the MTB value may be preferred, since countries such as the U.S. will (at least historically) have a larger portion of 'trapped cash' on their balance sheet than others due to tax considerations. Making deductions for this on the basis of a q definition defined using capital values only would then lead to problems. Traditional Tobin's q proxies must deduct all or most of the firm's cash to arrive at just the firm's operating assets. This may also create a strong time bias in Tobin's q measures for the U.S. (ibid.).

In addition, many firms in Compustat do not separate their assets into current and non-current assets, such as Berkshire Hathaway, required for a proper computation of Tobin's q.

This makes the MTB the least sensitive measure to differing accounting reporting requirements between and within countries. We compared several different measures of q across countries in our sample. The distribution of q as the MTB is most similar, and with a lower variance, across the Compustat Global and Compustat North America databases. Certain issues though will be present across all proxies for Tobin's Q. We would expect q values to vary greatly depending on the accounting rules used by the firm regarding revaluation of the market value of PPEGT. The ability to revalue assets (to fair value) under IFRS might create significant differences in the carrying value of assets as compared with US GAAP (PWC 2018; Gordon et al. 2008). While IFRS permits revaluation, US GAAP generally utilizes historical cost and prohibits revaluations of fixed capital. *As a result, a downward bias will be expected in book values of U.S. GAAP firms*. Compounding this is the fact that with US GAAP, reversal of impairment is prohibited, while under IFRS it is permitted. We would expect then that q values would be higher in the U.S. than in other advanced economies. This is exactly what we see in Table 11.

From a computational perspective, using a variable which can only take on positive have considerable benefits too - especially in a Bayesian model. This allows us to log the variable which makes the sampling process several times quicker. Secondly, it helps reduce heteroskedasticity considerably. This can be seen by running simple quantile investment regressions of q on investment and plotting the fits across quantiles (Koenker et al. 2001). Thirdly, q becomes lognormal when logged. This is related to Q being roughly log-normal. Finally, a log interpretation of q is empirically more sensible since in general q values tend to have quite a high variance (rather than in theory, where they are assumed to generally be between zero and one). A firm with a q value of 20 we would expect to react differently to a one unit change in its value than a firm with a q value of 0.5 or 1.

C.3 Country and Sample Selection and Categorisation

Country location of firm is based on foreign incorporation code (FIC) rather than country of headquarter or country of listing. We have 18 countries in total. Country selection is first based on average GDP per capita (nominal) US$ between 1994-2017 of $20,000 or more. To be included in the final sample the country needed to have 1,900 or more observations in the Compustat file between 1994-2020. This gives us 21 developed economies in our sample consisting of 18 countries, the U.S, and 2 major tax havens (Bermuda and Cayman Islands).

Advanced Economy (excluding U.S. and Japan): Includes Great Britain (“GBR” - 23,344 observations), Canada (“CAN” - 2,562), Australia ("AUS” - 15,007), Cayman Islands (“CYM” - 10,930), France (“FRA” - 10,898), Germany (“DEU” - 10,590), Singapore (“SG” - 9,252), Bermuda (“BM” - 8,566), Sweden (“SWE” - 6,556), Israel (“ISR” - 4,400), Switzerland ("CHE” - 3,958), Italy (“ITA” - 3,905), The Netherlands (“NLD” - 3,131), Norway (“NOR” - 2,715), Denmark (“DNK” - 2,169), New Zealand (“NZL” - 1,965), Spain (“ESP” - 2,245), and Finland (“FIN” - 2,542). USA consists of 104,531 observations and Japan (“JPN”) 56,868.

Table 6. Data Sample Summary

Country	1994-2001	2002-2007	2008-2020
USA	41,554	24,121	38,856
Advanced Economies	23,394	30,899	73,372
Japan	6,985	15,510	34,373

Note: *Showing number of data points in our sample, by year and country grouping. Shows shrinking number of new lists in the U.S. Tax haven country firms are included with advanced economies.*

Table 7. Sample Size Breakdown

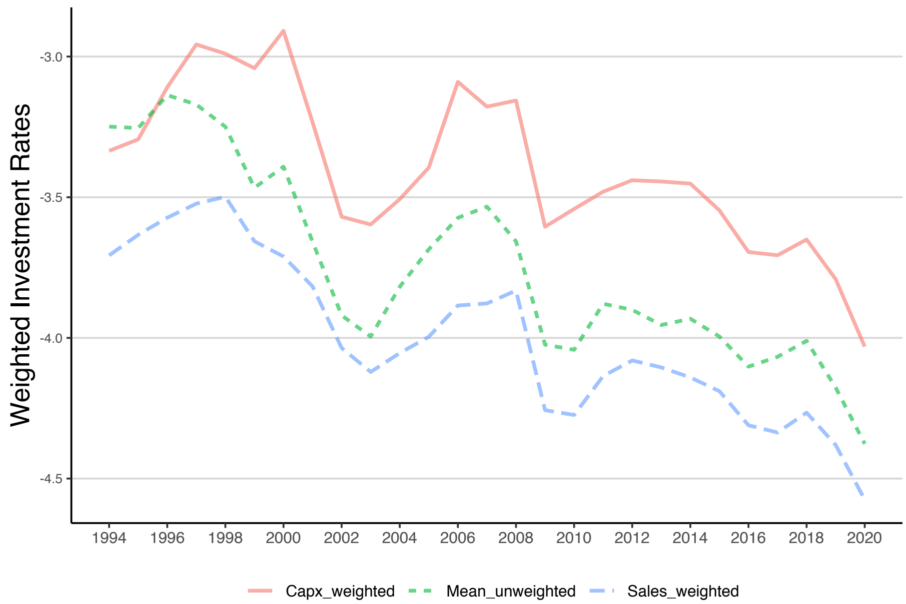
Country	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
USA	4,830	5,258	5,644	5,604	5,366	5,250	5,014	4,588	4,337	4,150	4,071	3,955
Advanced Economies	1,455	1,714	2,344	2,956	3,194	3,486	3,939	4,306	4,487	4,683	4,951	5,341
Japan	30	31	33	32	31	1,895	2,444	2,489	2,516	2,536	2,573	2,617

Country	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
USA	3,849	3,759	3,507	3,363	3,269	3,149	3,074	3,077	3,102	2,979	2,835	2,743
Advanced Economies	5,630	5,807	5,465	5,461	5,407	5,433	5,432	5,465	5,539	5,659	5,708	5,872
Japan	2,648	2,620	2,603	2,496	2,413	2,398	2,429	2,606	2,663	2,686	2,715	2,811

Country	2018	2019	2020
USA	2,653	2,579	2,526
Advanced Economies	5,985	6,153	5,793
Japan	2,837	2,870	2,846

B. Descriptive Statistics

Figure 12. Mean Investment Rate Weighted and Unweighted by Firm Size Proxies (log scale), 1994-2020



Note: *Mean investment rate: unweighted, weighted by sales, and weighted by capx. Indicates a fairly broad decline across different firm sizes.*

Table 8. Investment Rate by Country and Year Group

Country	Time Period	MAD	0.25	0.5	0.75	0.9	0.99
USA	1994-2001	0.06	0.04	0.07	0.13	0.24	0.63
USA	2002-2007	0.04	0.02	0.04	0.08	0.16	0.48
USA	2008-2020	0.03	0.02	0.04	0.07	0.14	0.43
Advanced Economies	1994-2001	0.05	0.04	0.07	0.11	0.20	0.58
Advanced Economies	2002-2007	0.04	0.03	0.05	0.09	0.20	0.72
Advanced Economies	2008-2020	0.04	0.02	0.04	0.08	0.15	0.54
Japan	1994-2001	0.03	0.02	0.03	0.06	0.09	0.23
Japan	2002-2007	0.03	0.02	0.03	0.06	0.10	0.24
Japan	2008-2020	0.03	0.02	0.03	0.05	0.09	0.26

Note: *Investment Rates decline over time secularly across the distribution of firms. Japan's investment rates, however, show a degree of consistency, albeit at very low levels.*

Table 9. Cash Flow Rate Percentiles by Country and Year Group

Country	Time Period	MAD	0.25	0.5	0.75	0.9	0.99
USA	1994-2001	0.12	-0.04	0.07	0.14	0.27	1.01
USA	2002-2007	0.12	-0.04	0.07	0.14	0.24	1.03
USA	2008-2020	0.10	-0.01	0.07	0.14	0.24	1.00
Advanced Economies	1994-2001	0.08	0.04	0.09	0.14	0.24	0.84
Advanced Economies	2002-2007	0.10	0.01	0.08	0.15	0.28	1.30
Advanced Economies	2008-2020	0.10	-0.00	0.07	0.14	0.26	1.32
Japan	1994-2001	0.04	0.04	0.06	0.10	0.15	0.45
Japan	2002-2007	0.05	0.04	0.07	0.11	0.18	0.80
Japan	2008-2020	0.05	0.04	0.07	0.12	0.25	1.48

Note: *MAD is the median absolute deviation. Cash flow rates increase are largely stable over time over time.*

Table 10. q Value Percentiles by Country and Year Group

Country	Time Period	MAD	0.25	0.5	0.75	0.9	0.99
USA	1994-2001	0.72	0.80	1.19	2.06	3.93	13.36
USA	2002-2007	0.77	0.87	1.29	2.14	3.80	13.64
USA	2008-2020	0.74	0.82	1.22	2.07	3.75	12.56
Advanced Economies	1994-2001	0.74	0.62	1.00	1.95	4.67	21.40
Advanced Economies	2002-2007	0.73	0.64	1.03	1.86	3.83	16.81
Advanced Economies	2008-2020	0.62	0.59	0.92	1.63	3.22	13.91
Japan	1994-2001	0.29	0.41	0.58	0.83	1.57	10.93
Japan	2002-2007	0.35	0.43	0.63	0.97	1.84	14.13
Japan	2008-2020	0.38	0.44	0.64	1.06	2.16	16.70

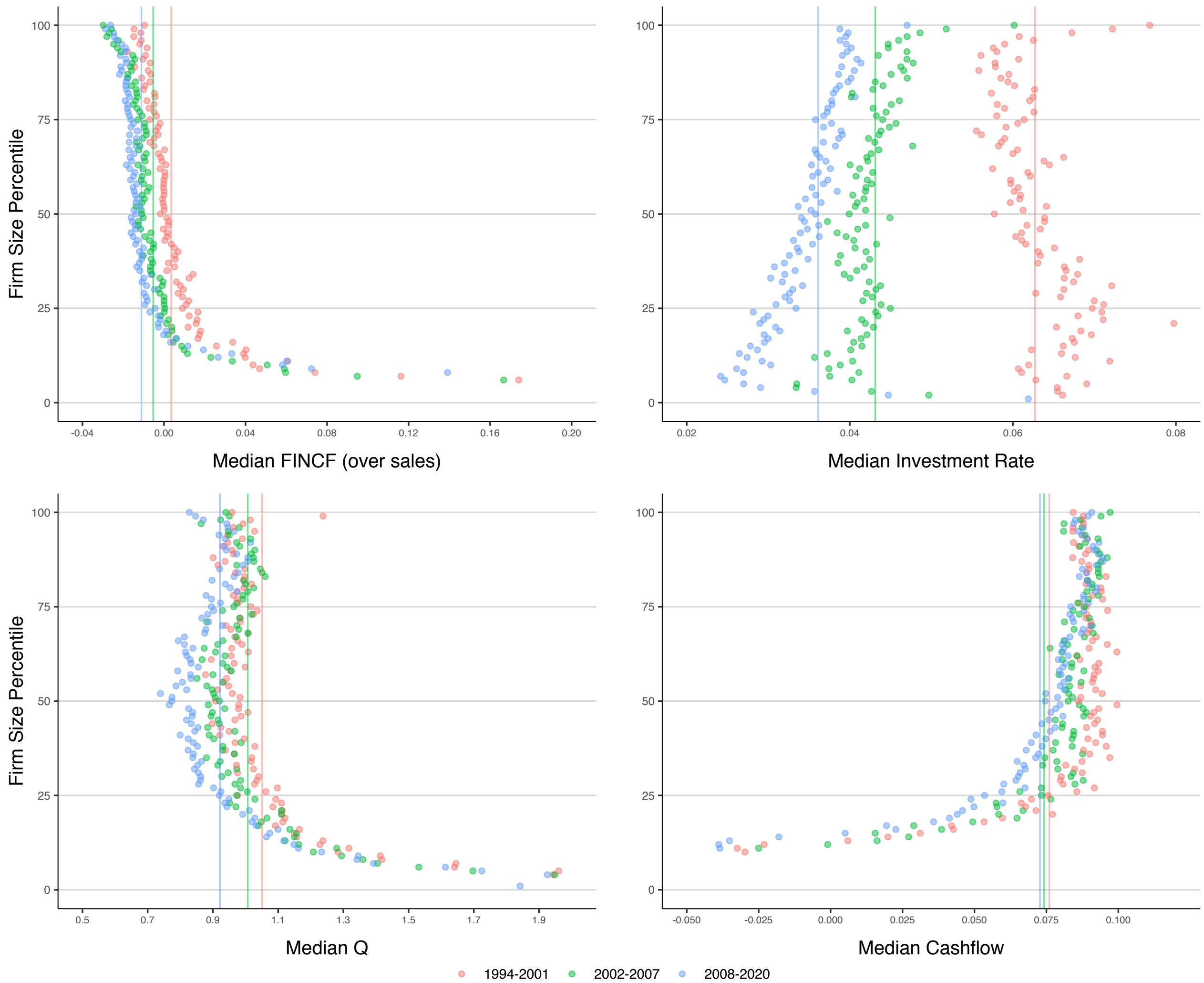
Note: *MAD is the median absolute deviation. q values stagnate or decline within advanced economies. Within the U.S. the movement in q values over time is cyclical and towards less variability (declining and the top increasing at the bottom). Japan shows a mild recovery in q values, but still far below other countries.*

Table 11. Summary Statistics of Tobin's Q by Country Group

Country	Min.	1st Qu.	Median	Mean	MAD.	3rd Qu.	Max.	
1	USA	0.14	0.82	1.22	1.97	0.74	2.08	35.96
2	Advanced Economies	0.13	0.61	0.96	1.82	0.66	1.74	36.08
3	Japan	0.13	0.43	0.63	1.25	0.36	1.00	36.05

Note: *MAD stands for 'median absolute deviation'. U.S. q values are higher and with greatest spread. High q values for U.S. firms are probably partly due to the downward bias over time in the book values of fixed capital under US GAAP methods. These do not allow for revaluation upward of fixed assets to fair value, or reversal of impairment charges.*

Figure 13. Key Variables by Firm Size and Time Period



Note: *Firm size based on gross fixed capital stock. Largest firm is 100th percentile. Vertical line is median value for that year period. Some values cut off for top and bottom percentiles to reduce graph scale. Smaller firms have higher rates of investment though the relationship is non-linear. Investment rates have declined for all firm sizes, especially for firms in the smallest bin size. Q values have decline the most for the largest firms, followed by medium size firms. The smallest firms have even see Q values increase. Cashflow rates have increased the most for smaller firms.*

B. Group Predictors and Regression

FINCF is defined as 'net external financing activities' and is one of the three primary cash flow statement balances. It records almost all cash inflows and outflows between the firm and its external creditors, bondholders, and shareholders,[60] and includes:[61]

- *Long-term debt issuance and principal repayments*[62]
- *Current debt issuance and principal repayments*
- *Cash dividends paid*
- *Purchase of common and preferred stock*
- *Sale of common and preferred stock*
- Other: *Debt and equity issuance costs, changes in stock options, minority shareholder dividends, dividends on subsidiary stock, and tax benefits of stock options.*

Figure 14. Corporate Secular Stagnation as Firms Become Net External 'Releasers' of Funds, 1994-2020

Note: *Japan, the classic example of secular stagnation, has since our series begins a negative net external corporate financing balance (bottom graphs) and the highest proportion of firms who are net external releasers of funds to shareholders, creditors, and bondholders (top graph). Advanced economies and the U.S. turn to a negative external financing balance around 2001. Red vertical line is for the year 2000. Net External corporate financing balance based on the median firm-level value for FINCF normalized by sales. Mean (i.e. aggregated) values for corporate sectors shows a similar trend, but is less robust. 'Percentage of releasers' is the proportion of total firms who have a negative FINCF value.*

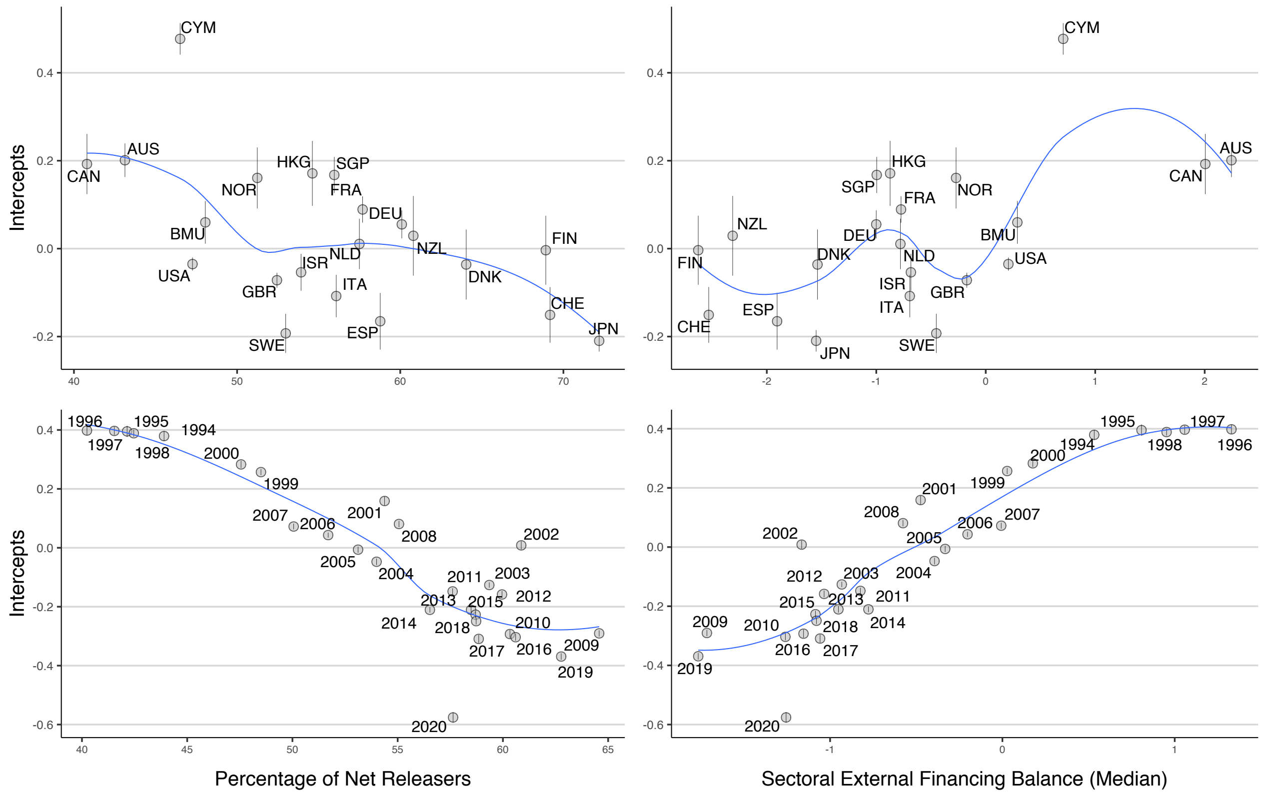
[60] It excludes dividend received. Dividend received is instead located in cash flow for North America firms.

[61] The definition below is for firms following U.S. GAAP accounting standards. Compustat Global firms instead tend to use IFRS accounting standards and so, define FINCF somewhat differently. IFRS permits interest and dividends received and paid, as well as bank overdrafts, to be classified as 'operating activities', 'investing activities', or under FINCF as 'financing activities'.

[62] FINCF excludes interest payments on debt. It includes the principal payments on capital (financial) lease liabilities, since a debt is being accumulated in order to gain an asset.

We use the two aggregate FINCF 'macro' predictors in Figure 14 to try and account for the secular slowdown in the impetus of firms to invest.

Figure 15. Estimated Mean Group Investment Rate Plotted Against Macroeconomic Predictors



Note: *Fitted LOESS line between the intercept coefficients (the data) and the two group variables used to account for differences in investment rates over time - i.e. secular stagnation of our pooled advanced economy sample. Non-linear fit is evident for the external financing balance of the corporate sector in the bottom right hand graph, indicating that too much borrowing might be counterproductive. CYM is an outlier. 2020 is an outlier as firms saw large net external injections of finance into the firm even as baseline investment rates fell heavily.*

The fit of our model is intuitively illustrated in Figure 15 using a simple local polynomial regression (LOESS). This shows the fitted regression (LOESS) line between the estimated intercept coefficients (the data) and the macroeconomic ('group-level') predictors. Although the regression we run only focuses on accounting for variation in the intercept across time, we can see that it would also be somewhat (though less) effective in accounting for variation in time-varying investment rate intercepts between countries. Most countries and years follow the predicted line very well. The U.S. stands out as having a lower intercept coefficient given its relatively low proportion of 'net releasers' and positive median sectoral balance (telling us that the median firm is a net external borrower of funds).

C. FINCF Variable

FINCF has the benefit of being widely reported by all firms and covers a number of items that are difficult to obtain individually in cross-country firm-level datasets, such as share repurchases and share issuances. As a 'net' variable it has the benefit of being largely invariant to transactions which only ultimately impact the firm's capital structure and which might mistakenly be taken as a sign of financial distress, or financial slack (Gutiérrez et al. 2017b; Farre-Mensa et al. 2016).[63]

FINCF is similar to the net external financing measure used in Gutiérrez et al. (2017b), drawing on Frank et al. (2008), but includes short-term borrowing. We propose that FINCF increasingly reflects the fact that firms have a surplus of available financing relative to diminishing investment opportunities (Appendix C.3). That such a relative surplus would manifest itself through firms releasing more funds, in net, is not true by definition since, following the cash flow statement identity, a surplus of funds relative to capital expenditure can instead be met through the firm increasing its net purchase of financial assets, or through a net increase in the retention of funds.

A similar version of our 'net external financing activities' variable is used by Frank et al. (2003) for a Pecking Order test of firms' debt structure. Our variable is calculated differently, though.[64] Gutiérrez et al. (2017b, Fig. 15), drawing on Frank et al. (2003), explore why the investment slowdown in the U.S. is most pronounced among firms with high credit ratings (those rated AA to AAA) compared to firms with lower credit ratings (those rated below AA).[65] They come up with several important empirical findings which support our conclusions.[66]

[63] For example, if a firm repurchases $20 million dollars of its shares while at the same time issuing corporate debt worth $20 million, then we propose that all that has happened is that the firm's capital structure has become more leveraged.

[64] Frank et al. (2003) do not include dividends paid with net equity issuance though, as our variable does, following GAAP and IFRS guidelines. Dividends are instead part of the firm's 'financing deficit', while changes in short-term debt — i.e. Compustat item DLCCH. — are entirely excluded.

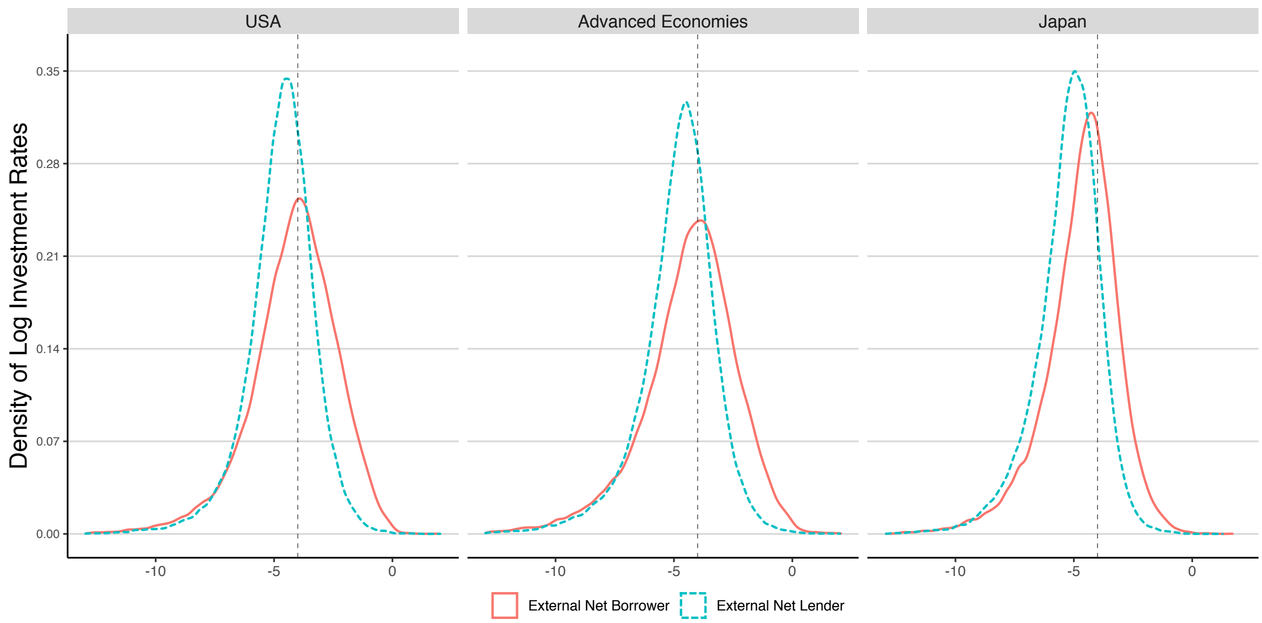
[65] They calculate the firm's 'financing deficit' as roughly equal to (FINCF), but they do not include changes in short-term debt or dividends.

[66] They find: (1) More highly rated firms turned to an external financing surplus around 1990, while this happened much later (mid-2000s) for less highly rated firms; (2) The shift towards negative external financing — i.e. net 'releaser' of funds — has empirically been driven by negative net equity issuance (the sale and purchase of common and preferred stock), since long-term net debt issuance has remained positive; (3) Moreover, net debt issuances have been *positive* for firms with high credit ratings, and have run concurrently to large *negative* net equity issuance by this same group of firms since the mid-1980s. This is exactly what Agency Theory might recommend for cash-rich firms facing a secular stagnation environment; and (4) Even firms with worse credit ratings, and with large positive net debt issuance, have had negative equity issuance since the mid-1980s. This highlights the limitations of using gross distributions to shareholders as a measure of financial constraints. Together, these findings support our secular stagnation hypothesis, despite using a related definition only, since the trend towards disgorging cash externally is driven by financially healthier firms engaging in (negative) net equity issuance, even as their net debt issuance remains positive — and increasing.

C.1 FINCF Visual Description

Figure 16 shows a clear difference in the distribution of investment rates for firms who are net external borrowers of funds and firms who are net external releasers of funds. This indicates that it is not just differences in surplus cash flow rates driving this trend but also structurally weaker investment opportunities for these firms. We confirm this in the 'DNA' graphs below.

Figure 16. Investment Rates by FINCF and Country Grouping

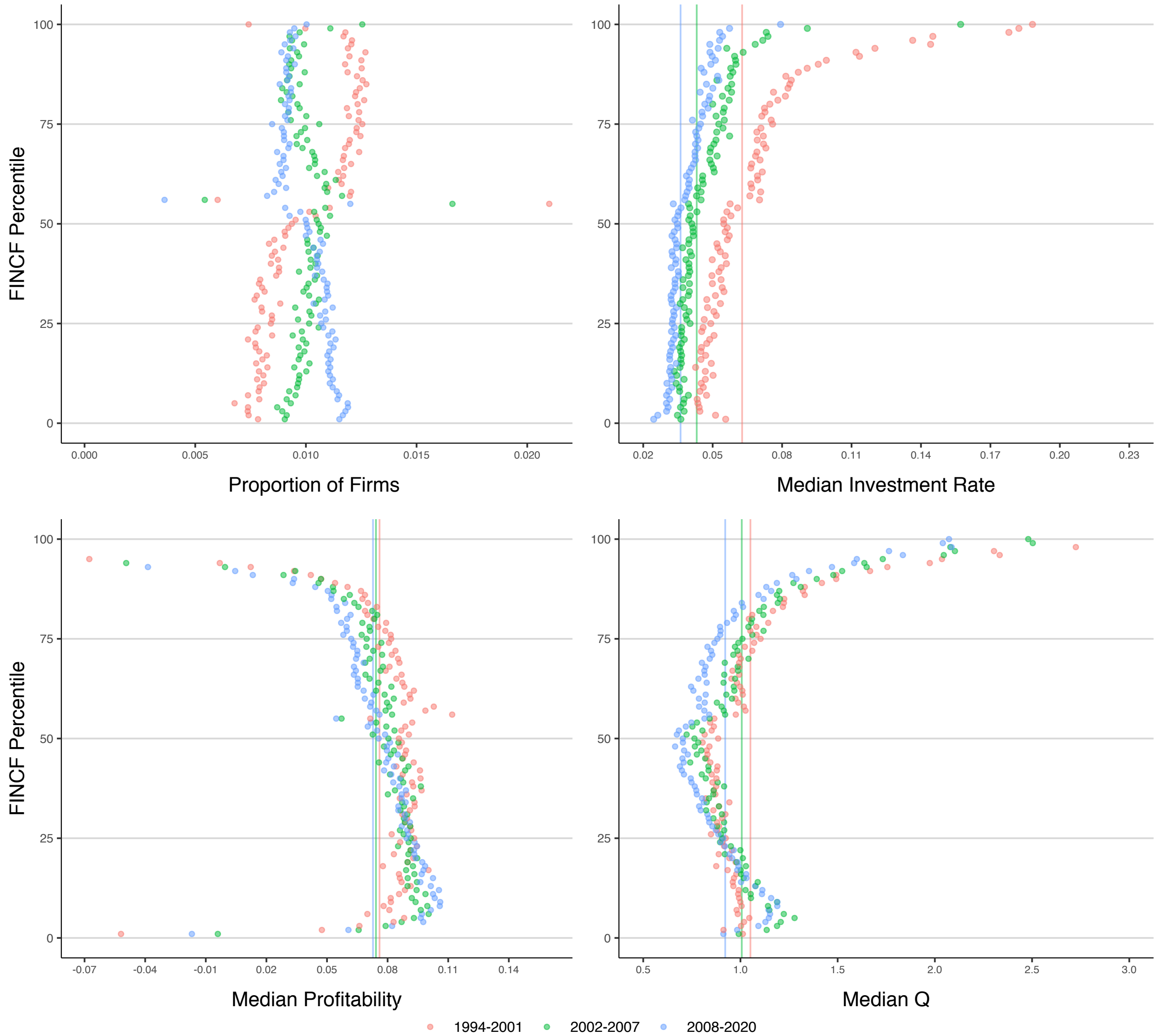


Note: *Kernel density approximation of firm-level investment rates on log2() scale. Firms' investment rates are closely tied to their net external financing positions. Firms that are net external 'releasers' of funds have a median investment rate of 3.7% (.03 MAD), compared to an investment rate of 5.6% (.056 MAD) for firms that are net external 'borrowers'. As more firms in the economy become net external 'releasers' of funds, economy-wide investment rates should slow.*

The DNA graphs in Figure 17 show changes in our key quantities of interest over time by FINCF percentile and country grouping. Each DNA dot ('atom') shows the median value of the variable in question for a specific FINCF firm percentile (unless stated otherwise). While the different coloured strands reflect different time periods. Strands loosen or tighten over time. Vertical lines for each time period show the median pooled value. These vertical lines show that median cash flow rates for our pooled sample have increased even as raw Q values and investment rates have declined. The DNA graphs allow us to explore this variation in greater detail across the FINCF bins.

We see an increase over time in the proportion of total firms that are large net external releasers of funds (percentiles 50 $\rightarrow$ 0) and a decline in the proportion of firms that are net external borrowers (percentiles 50 $\rightarrow$ 100). This is reflected in a shifting out - an increase - in the proportion of mid-tier FINCF firms (percentiles 75 $\rightarrow$ 10), but a shift in (decrease in the proportion of) the largest net

Figure 17. Cash Flow, Q, Investment Rate, and proportion of observations by FINCF Bin.



Note: *Large net external releasers of funds are smaller percentiles* $50 \to 0$ *(bottom of y-axis); firms that are net external borrowers tend to be percentiles* $50 \to 100$ *(top of y-axis). Bin widths calculated on pooled unstratified sample. The largest net releaser and also large net borrowers tend to be cash flow poor.*

releasers of funds (percentiles 10 $\rightarrow$ 0) and the largest net borrowers of funds (percentiles 10 $\rightarrow$ 0).

With respect to investment opportunities (bottom right hand graph Figure 17): FINCF seems to capture a stable relationship across countries and firms in firms' underlying investment opportunities. The relationship between q and FINCF percentile is non-linear. Firms that borrow the most or release the most have more investment opportunities than firms in the middle. (The main difference between these two types of firms is their degree of cash flow: borrowers have negative or low cash flow rates while releasers have high positive cash flow rates.) Median q values have in general shifted inwards for advanced economy firms over time (from above 1 to below 1). Interestingly q values have increased for the top 15 or so FINCF releasing percentiles in advanced economies. Some values cut off for top and bottom percentiles to reduce graph scale.

Note that, as per Figure 17 (top right hand graph), investment rates tend to be higher for firms that borrow more and release less (percentiles 50 $\rightarrow$ 100). Investment rates have shifted inwards for all percentiles across all 3 time periods for advanced economy firms, from 6.5% to 4.5% to 4%. They have declined the most for firms that are larger net borrowers (percentiles 100 $\rightarrow$ 50)) of external funds. (This is unlikely to reflect a growing financial constraint since these firms have also had the largest increase in cash flow rates over time - see following graph.)

Median cash flow rates are highest (around 14%) for firms that release the most (Figure 17 bottom left hand graph), declining constantly and lowest for firms that borrow the most (negative for around the top 5 percentiles - values not able to fit on graph's scale). Profitability tend to be higher for larger more mature firms so this will be reflected in the above too. Median cash flow rates have shifted upward over time, especially for advanced economy firms that tend to borrow the most (percentiles 75 $\rightarrow$ 100) and for firms that release the most (top 5 percentiles). They have increased from 6.8% to 7.9% to 8.7% post-crisis for advanced economy firms. That this has gone hand-in-hand for advanced economy firms with lower q values points to the role of higher profit margins in higher cash flow rates. Some values cut off for top and bottom percentiles to reduce graph scale.

C.2 Alternative Explanations for FINCF and the Other Cash Flow Statement Items

Is the positive observed relationship between FINCF and investment not simply a result of a 'debt-overhang' (Myers 1977)? Our sample shows signs of de-leveraging by firms in several countries consistent with a debt-overhang. This could provide a compelling narrative if it leads to firms paying off principal debt, resulting in a negative FINCF, and increasing savings (or retention out of cash flow) to fund debt repayment rather than reinvestment (Koo 2011).[67] In addition, median leverage levels (defined as short-term plus long-term debt over equity) are much higher for net external 'borrowers' than net external 'releasers' of funds (roughly double).[68] This implies that leverage levels are probably declining over time for most firms. The fit of this to our data is weak though. Firstly, the level of median leverage at < 0.5 is not high. Secondly, the decline in leverage is evident across the distribution of firms in both advanced economies and developing economies, despite their very different investment rate trends. Moreover, no decline in leverage is evident for U.S. firms, except during 2002-2007 or so. The latter is consistent with the findings by Gutiérrez et al. (2017b) that U.S. firms have been positive issuers of net debt, including highly credit-worthy firms. Thirdly, the number of firms in our sample experiencing a balance sheet recession, proxied by 'negative equity', never goes above $\approx 4.5\%$, such that they are unlikely to have a notable impact. Fourthly, the relationship between leverage and investment in our sample is complex and weak: Leverage levels are highest among developing economy firms, but also declining most strongly for them. These firms also have higher rates of investment, even though the literature tends to find that firms with lower debt burdens should invest less (Stein 2003).

Lastly, studies increasingly focus on the corporate sector shifting from being 'net borrowers' to being 'net lenders' in the national accounts (NA). This is linked either to increased savings (Armenter et al. 2017; P. Chen et al. 2017), or decreased investment (Gruber et al. 2015). The national account concept of net lending is defined as Savings (profits less dividends) – Investment. As such, these findings, while generally supportive of ours, are not directly comparable for several important reasons: Firstly, corporate net lending in the NA is highly sensitive to how activities in other sectors of the economy are classified (Ruggles 1993). Secondly, the NA concept only shows what firms are *able* to lend (or borrow) based on movements in the sectoral flows of retained profits relative to investment expenditure. It does not indicate what firms are *actually doing*. Thirdly, it also does not indicate what firms are *able to do*. This would require taking into account how a firm's cash and other stocks impact its financial constraints. The NA effectively ignores share repurchases from its concept of 'net lending', since it is treated as a use of funds rather than a prior deduction from profits to arrive at savings, or retained

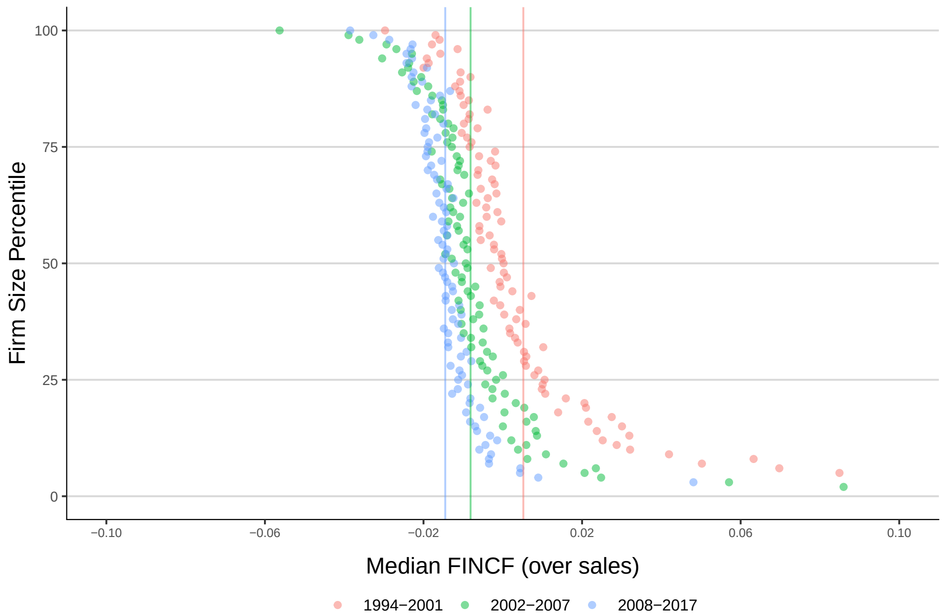
[67] Even though CAPX out of cash flow has not declined at the median in the U.S. since the 2000s, and in fact has even increased.

[68] Firms that are very large net external 'borrowers' of funds, tend to have very low leverage levels, though, since they are young firms. This makes sense, since firms with low and negative levels of cash flow are almost always net 'borrowers' of external funds, while firms with high levels of cash flow are net 'releasers' of funds.

earnings. The NA concept also excludes share and debt issuances, since this is again a use of funds rather than a change in the firm's profits and retained earnings. As such, the concept gives us no real indication of firms' overall — i.e. net — financing demand, financing constraint, or actual behaviour. It is merely an accounting identity.

C.3 Life Cycle of the Firm and FINCF

Figure 18. FINCF by Firm Size



Note: *Net external 'releasing' (negative x-axis values) and net external dispersing of funds (positive x-axis values) tends to follow the life cycle of the firm: smaller firms (*0 → 50 *on y-axis) in their infancy with plenty of investment opportunities but negative cash flow borrow more (relative to sales), while larger (*50 → 100*), mature, firms tend to release more as their investment opportunities tend to fall short of their by now large cash flow rates. Some values cut off for top and bottom percentiles to reduce graph scale.*

Does the above observed pattern in FINCF not reflect simply the life cycle of the firm? As firms mature and relative investment opportunities dry up firms tend to distribute more surplus (H. DeAngelo, L. DeAngelo, and Stulz 2006; H. DeAngelo, L. DeAngelo, Skinner, et al. 2009; Damodaran 2010).[69] Figure 18 shows that firms' net external financing flow position follows the firm's life cycle (proxied by its size) quite closely: younger firms have larger investment opportunities relative to their low or negative cash flow, as a result they borrow substantially relative to sales (large and positive FINCF). While more mature firms with fewer investment opportunities (including relative to a large and positive cash flow) land up distributing in net their excess surplus, resulting in a large negative FINCF. We see a very similar shape and tendency if we instead used deflated firm capital stock percentiles as the y-axis variable.

[69]The shortening of firms' life cycle may be speeding this up (Damodaran 2015).

This raises the question of whether the trends in investment rates and FINCF is simply a Compustat sample issue, i.e. average firm age increasing in Compustat. This is unlikely. Firstly, the growing trend towards firms' engaging in less borrowing and more dispersing of funds externally is a feature across all firm sizes in advanced economies. This is unsurprising since the increase in cash flow rates and the decline in investment opportunities is a feature across all firm sizes. Secondly, Table 6 shows that firms in our non-U.S. sample do not display this same decline in public listing as in the U.S.. This decline begins in 1996 in our specific U.S. sample. Outside of the U.S. listing have not been declining until more recently (Doidge et al. 2018; Piwowar 2019). Thirdly, it is possible that the firm's life cycle has simply become compressed (Damodaran 2015). This would account for the shift across all firm sizes in FINCF. However, this seems to largely be a feature of 'technology' firms (loosely defined), which are only a small portion of our total sample of firms.